8.6 To cool a given room it is necessary to supply 4 ft³/s of air through an 8-in.-diameter pipe. Approximately how long is the entrance length in this pipe?

\[ V = \frac{Q}{A} = \frac{4 \frac{\text{ft}^3}{\text{s}}}{\frac{\pi}{4} \left( \frac{8}{2} \text{ ft} \right)^2} = 11.5 \frac{\text{ft}}{\text{s}} \]

Thus, with \( V = 1.57 \times 10^{-4} \frac{\text{ft}^2}{\text{s}} \) (see Table 1.6)

\[ \text{Re} = \frac{V D}{\nu} = \frac{11.5 \frac{\text{ft}}{\text{s}} \left( \frac{8}{2} \text{ ft} \right)}{1.57 \times 10^{-4} \frac{\text{ft}^2}{\text{s}}} = 48,800 > 4000 \] so the flow is turbulent.

Hence,

\[ \frac{L_e}{D} = 4.4 \text{Re}^{1/6}, \quad \text{or} \quad L_e = 4.4 \left( 48,800 \right)^{1/6} \left( \frac{8}{2} \right) = 17.7 \text{ ft} \]

8.7 The pressure drop needed to force water through a horizontal 1-in.-diameter pipe is 0.60 psi for every 12-ft length of pipe. Determine the shear stress on the pipe wall. Determine the shear stress at distances 0.3 and 0.5 in. away from the pipe wall.

For a horizontal pipe \( \frac{\Delta P}{L} = \frac{2\tau}{R} \) or \( \tau = \frac{2 \Delta P}{L R} \)

Thus,

\[ \tau = \frac{2 \left( 0.6 \times 144 \frac{\text{lb}}{\text{ft}^2} \right)}{2 \left( 12 \text{ ft} \right)} = 3.6 \frac{\text{lb}}{R \text{ ft}^2} \], where \( R \sim \text{ft} \)

Hence,

\[ \tau_w = 3.6 \left( \frac{0.5}{12} \right) = 0.15 \frac{\text{lb}}{\text{ft}^2} \]

and with \( r = (0.5 - 0.3) \text{ in.} = 0.2 \text{ in.} \),

\[ \tau = 3.6 \left( \frac{0.2}{12} \right) = 0.06 \frac{\text{lb}}{\text{ft}^2} \]

Finally, with \( r = (0.5 - 0.5) \text{ in.} = 0 \text{ in.} \), \( \tau = 0 \)

8-3
8.9 Water flows in a constant diameter pipe with the following conditions measured: At section (a) \( p_a = 32.4 \text{ psi} \) and \( z_a = 56.8 \text{ ft} \); at section (b) \( p_b = 29.7 \text{ psi} \) and \( z_b = 68.2 \text{ ft} \). Is the flow from (a) to (b) or from (b) to (a)? Explain.

Assume the flow is uphill. Thus, \( \frac{p_a}{g} + \frac{V_a^2}{2g} + z_a = \frac{p_b}{g} + \frac{V_b^2}{2g} + z_b + h_L \)

or with \( V_a = V_b \),

\[ h_L = \frac{p_a}{g} + z_a - \frac{p_b}{g} - z_b = \frac{(32.4 \text{ psi} - 29.7 \text{ psi})(144 \text{ ft}^3)}{62.4 \frac{\text{lb}}{\text{ft}^3}} + 56.8 \text{ ft} - 68.2 \text{ ft} \]

or \( h_L = -5.17 \text{ ft} < 0 \), which is impossible. Thus, the flow is downhill, from (b) to (a).

8.10 A fluid of specific gravity 0.96 flows steadily in a long, vertical 1-in.-diameter pipe with an average velocity of 0.50 ft/s. If the pressure is constant throughout the fluid, what is the viscosity of the fluid? Determine the shear stress on the pipe wall.

Assume laminar, calculate \( \mu \), then check to determine if \( Re < 2100 \). For laminar flow down the pipe \( (\theta = -90^\circ) \) with constant pressure (\( \Delta p = 0 \))

\[ V = \frac{(\Delta p - \rho g \sin \theta) D^2}{32 \mu} \]

or \( \mu = \frac{0.96(62.4 \frac{\text{lb}}{\text{ft}^3})(\frac{1}{2} \text{ ft})^2}{32 (0.5 \frac{\text{ft}}{4})} = 0.0260 \frac{\text{lb}\cdot\text{s}}{\text{ft}^2} \)

Note:

\[ Re = \frac{\rho V D}{\mu} = \frac{0.96(1.94 \frac{\text{slug}}{\text{ft}^3})(0.5 \frac{\text{ft}}{4})(\frac{1}{2} \text{ ft})}{0.0260 \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}} = 2.98 < 2100 \]

Thus, the flow is laminar as assumed and \( \mu = 0.0260 \frac{\text{lb}\cdot\text{s}}{\text{ft}^2} \)

Also, with \( \Delta p = 0 \), \( \Sigma F_x = 0 \) gives \( \pi Dl T_w = \frac{\rho V D^2}{2} \), or

\[ T_w = \frac{\rho V D^2}{4} = 0.96(62.4 \frac{\text{lb}}{\text{ft}^3})(\frac{1}{2} \text{ ft}) = 1.25 \frac{\text{lb}}{\text{ft}^2} \]
8.14 Oil of $SG = 0.87$ and a kinematic viscosity $\nu = 2.2 \times 10^{-4} \text{ m}^2/\text{s}$ flows through the vertical pipe shown in Fig. P8.14 at a rate of $4 \times 10^{-4} \text{ m}^3/\text{s}$. Determine the manometer reading, $h$.

\[ V = \frac{Q}{A} = \frac{4 \times 10^{-4} \text{ m}^3}{\pi \frac{h}{4} (0.02 \text{ m})^2} = 1.27 \frac{\text{ m}^2}{\text{s}} \text{ so that} \]

\[ Re = \frac{\rho V D}{\mu} = \frac{V D}{\nu} = \frac{(1.27 \frac{\text{ m}^2}{\text{s}}) (0.02 \text{ m})}{2.2 \times 10^{-4} \frac{\text{ m}^2}{\text{s}}} = 115 < 2100 \]

The flow is laminar with

\[ Q = \frac{\pi (\Delta \rho + \gamma l) D^4}{128 \mu l}, \text{ or } \Delta \rho = \frac{128 \mu l Q}{\pi D^4} - \gamma l \]

Hence, with $\gamma = SG \gamma_0 = 0.87 (9.81 \frac{\text{ kN}}{\text{ m}^3}) = 8.53 \frac{\text{ kN}}{\text{ m}^3}$ and

\[ \mu = \nu SG (\gamma_0 = (2.2 \times 10^{-4} \frac{\text{ m}^3}{\text{s}}) (0.87 \times 100 \frac{\text{ kg}}{\text{ m}^3}) = 0.191 \frac{\text{ N} \cdot \text{s}}{\text{ m}^2} \]

Eq. (1) gives

\[ \Delta \rho = \frac{128 (0.191 \frac{\text{ N} \cdot \text{s}}{\text{ m}^2}) (4 \text{ m})(4 \times 10^{-4} \frac{\text{ m}^3}{\text{s}})}{\pi (0.02 \text{ m})^4} - (8.53 \frac{\text{ kN}}{\text{ m}^3})(4 \text{ m})(10^3 \frac{\text{ N}}{\text{ kN}}) \]

or

\[ \Delta \rho = 4.37 \times 10^4 \frac{\text{ kN}}{\text{ m}^2} = 43.7 \frac{\text{ kN}}{\text{ m}^2} \]

Eq. (2)

From manometer considerations

\[ \rho_1 + \delta h_1 - \delta_m h + \delta h_2 = \rho_2, \text{ where } \delta_m = SG \delta_{\gamma_0} = 1.3 (9.81 \frac{\text{ kN}}{\text{ m}^3}) = 12.74 \frac{\text{ kN}}{\text{ m}^3} \]

and

\[ h_i = h - h_2, \text{ or } h_2 + h_i = h_i \]

Thus,

\[ \rho_1 - \rho_2 = \Delta \rho = \gamma (h_2 + h_i) + \delta_m h = (\delta_m - \gamma) h - \gamma l \]

Eq. (3)

Combine Eqs. (2) and (3) to give

\[ 43.7 \frac{\text{ kN}}{\text{ m}^2} = (12.74 - 8.53) \frac{\text{ kN}}{\text{ m}^3} h - (8.53 \frac{\text{ kN}}{\text{ m}^3})(4 \text{ m}) \]

or

\[ h = 18.5 \text{ m} \]
8.15 Determine the manometer reading, $h$, for Problem 8.44 if the flow is up rather than down the pipe.

\[ V = \frac{Q}{A} = \frac{4 \times 10^{-4} \text{m}^3}{0.02 \text{m}^2} = 1.27 \text{m} \text{s} \text{ so that } \]

\[ \text{Re} = \frac{\rho V D}{\mu} = \frac{V D}{\mu} = \frac{(1.27 \text{m}) (0.02 \text{m})}{2.2 \times 10^{-4} \text{m}^2/\text{s}} = 115 < 2100 \]

The flow is laminar with

\[ Q = \frac{\pi (\rho_1 - \rho_2) D^4}{128 \mu L}, \text{ or } \Delta \rho = \rho_1 - \rho_2 = \frac{128 \mu L Q}{\pi D^4} + \delta l \]

Hence, with $\gamma = \text{SG}_{h_2} = 0.87(9.81 \text{ kN/m}^3) = 8.53 \text{ kN/m}^3$ and

\[ \mu = \nu = \text{SG} (h_2) = (2.2 \times 10^{-4} \text{m}^2/\text{s})(0.87)(1000 \text{ kN/m}^3) = 0.191 \text{ N} \text{s/m}^2 \]

Eq. (1) gives

\[ \Delta \rho = \frac{128 (0.191 \text{ N} \text{s/m}^2)(4 \text{m})(4 \times 10^{-4} \text{m}^3)}{128 \text{N} \text{s/m}^2} + (8.53 \text{ kN/m}^3)(4 \text{m})(10^3 \text{ kN}) \]

or

\[ \Delta \rho = 1.19 \times 10^5 \text{ N/m}^2 = 111.9 \text{ kN/m}^2 \]

From manometer considerations

\[ \rho_1 - \delta h_1 + \rho_m h - \delta h_2 = \rho_2, \text{ where } \delta_m = \text{SG}_{h_2} - 1.3(9.81 \text{ kN/m}^3) = 12.74 \text{ kN/m}^2 \]

and $h_2 = l + h - h_1$ or $h_2 + h_1 = l + h$

Thus,

\[ \rho_1 - \rho_2 = \delta(h_2 + h_1) - \delta_m h = - (\delta_m - \delta) h + \delta l \]

Combine Eqs. (2) and (3) to give

\[ 111.9 \text{ kN/m}^2 = -(12.74 - 8.53) \text{ kN/m}^2 h + 8.53 \text{ kN/m}^2 (4 \text{m}) \]

or

\[ h = -18.5 \text{ m} \]

Note: Since $h < 0$ the manometer is displaced in the direction opposite that shown in the original figure.
8.42 Water flows steadily through the 0.75-in. diameter galvanized iron pipe system shown in Video V8.6 and Fig. P8.42 at a rate of 0.020 cfs. Your boss suggests that friction losses in the straight pipe sections are negligible compared to losses in the threaded elbows and fittings of the system. Do you agree or disagree with your boss? Support your answer with appropriate calculations.

\[ \text{Major loss} = f \frac{l}{D} \frac{V^2}{2g} \text{ where} \]
\[ l = (6 + 6 + 4 + 1) \text{ in.} = 17 \text{ in., } D = 0.75 \text{ in.} \]
\[ V = \frac{Q}{A} = \frac{0.02 \frac{\text{ft}^3}{\text{s}}}{\frac{\pi}{4} (0.75/12)^2 \text{ft}^2} = 6.52 \frac{\text{ft}}{\text{s}} \]

Thus, with
\[ \text{Re} = \frac{VD}{\nu} = \frac{6.52 \frac{\text{ft}}{\text{s}} (0.75/12 \text{ ft})}{1.21 \times 10^{-5} \frac{\text{ft}^2}{\text{s}}} = 3.37 \times 10^4 \text{ and} \]
\[ \frac{E}{D} = \frac{\nu}{C} = 8 \times 10^{-3} \text{ (see Table 8.1) we obtain (see Fig. 8.10)} \]
\[ f = 0.038 \text{ so that } f \frac{l}{D} \frac{V^2}{2g} = 0.038 \frac{17 \text{ in.}}{0.75 \text{ in.}} \frac{V^2}{2g} = 0.861 \frac{V^2}{2g} \]  \hspace{1cm} (1)

Also,
\[ \text{Minor loss} = \sum K_2 \frac{V^2}{2g} = [2(1.5) + 2 + 0.15] \frac{V^2}{2g} = 5.15 \frac{V^2}{2g} \]  \hspace{1cm} (2)

Thus, from Eqs. (1) and (2):
\[ \frac{\text{major loss}}{\text{minor loss}} = \frac{0.861 \frac{V^2}{2g}}{5.15 \frac{V^2}{2g}} = 0.167 = 16.7\% \]

Probably disagree with boss because pipe friction is about 17\% of other losses.
Air, assumed incompressible, flows through the two pipes shown in Fig. P8.60. Determine the flow rate if minor losses are neglected and the friction factor in each pipe is 0.020. Determine the flow rate if the 0.5-in.-diameter pipe were replaced by a 1-in.-diameter pipe. Comment on the assumption of incompressibility.

\[
\begin{align*}
\text{Flow Rate} &= \frac{\rho \nu}{\theta} + \frac{V^2}{2g} + z = h_1 + h_2 + \frac{\rho \nu^2}{2g} + z_3, \quad \text{where} \quad V_0 = 0, \quad z_0 = z_3, \quad \rho_0 = 0, \\
V_2 &= V_3, \quad h_4 = f_1 \frac{D_1}{D_2} \frac{V^2}{2g}, \quad h_2 = f_2 \frac{D_1}{D_2} \frac{V^2}{2g}, \quad \text{and} \quad V_1 = \frac{A_1}{A_2} = V_3 (\frac{D_2}{D_1})^2 = (\frac{0.5 \text{ in.}}{1.0 \text{ in.}})^2 V_2 = 0.25 V_2.
\end{align*}
\]

Thus, Eq. (1) becomes

\[
\begin{align*}
\rho \nu &= f_1 \frac{D_1}{D_2} \frac{(0.25 V_2)^2}{2g} + f_2 \frac{D_1}{D_2} \frac{V^2}{2g} + \frac{V^2}{2g}, \\
\rho &= \frac{1}{2} \rho \nu \left[ f_1 \frac{D_1}{D_2} (0.25)^2 + f_2 \frac{D_1}{D_2} + 1 \right]
\end{align*}
\]

With \( \rho_0 = \rho_0 R T_0 \) or \( \rho_0 = \frac{\rho_0}{R T_0} = \frac{(0.5 \text{ lb/ft}^3 + 14.7 \text{ psi}) (144 \text{ lb/ft}^3)}{(1716 \text{ ft-lb/slug} \cdot \text{lb})(150+460) \text{ deg F}} = 0.00209 \text{ slug/ft}^3 \) and \( f_1 = f_2 = 0.020 \), Eq. (2) gives

\[
(0.5 \text{ lb/ft}^3)(144 \text{ lb/ft}^3) = \frac{1}{2} (0.00209 \text{ slug/ft}^3) V_2 \left[ (0.0200)(\frac{20 \text{ ft}}{12 \text{ ft}})(0.25)^2 + \frac{20 \text{ ft}}{12 \text{ ft}} \right] + 1
\]

or \( V_2 = 79.5 \text{ ft/s} \) Thus, \( Q = A_2 V_2 = \frac{\pi}{4} \left( \frac{1}{12 \text{ ft}} \right)^2 (79.5 \text{ ft/s}) = 0.108 \text{ ft}^3/ \text{s} \)

If both pipes were 1 in. diameter, then \( V_1 = V_4 \) and Eq. (1) becomes

\[
\rho_0 = \frac{1}{2} \rho \nu \left[ f_1 \frac{D_1}{D_2} + f_2 \frac{D_1}{D_2} + 1 \right] \quad \text{or} \quad f_1 = f_2 = 1, \quad D_1 = D_2 = D_3
\]

Hence,

\[
(0.5 \text{ lb/ft}^3)(144 \text{ lb/ft}^3) = \frac{1}{2} (0.00209 \text{ slug/ft}^3) V_2 \left[ 0.02 \left( \frac{40 \text{ ft}}{12 \text{ ft}} \right) + 1 \right]
\]

or \( V_2 = 80.6 \text{ ft/s} \) Thus, \( Q = A_2 V_2 = \frac{\pi}{4} \left( \frac{1}{12 \text{ ft}} \right)^2 (80.6 \text{ ft/s}) = 0.440 \text{ ft}^3/ \text{s} \)

Since \( \rho = \rho R T \), it follows that

\[
\frac{\rho_3}{\rho_0} = \frac{(f_2 \frac{D_1}{D_2})}{(f_2 \frac{R T_0}{T_0})} = \frac{f_2}{f_0} \frac{T_0}{T_3}
\]

If we assume \( T_3 = T_0 \) (it probably will not be, but it should be a reasonable approximation), then

\[
\frac{\rho_3}{\rho_0} \approx \frac{f_2}{f_0} = \frac{(14.7 \text{ psi})}{(0.5 \text{ psi})} = 0.967
\]

The flow is nearly incompressible.
(a) Derive an equation for \( h(t) \)

**1D Energy Eqn.**

\[
\frac{v_1^2}{2g} + z_1 + \frac{p_1}{\rho} = \frac{v_2^2}{2g} + z_2 + \frac{p_2}{\rho} + h_L
\]

Conservation of Mass \( \Rightarrow \)

\[
\frac{v_1 A}{2} = \frac{v_2 A}{2} = \sqrt{\frac{\pi D^2}{4}} \Rightarrow v_1 = v_2 = \sqrt{\frac{\pi D^2}{4A}}
\]

Thus the 1D Energy Eqn reduces to

\[h_L = z_1 - z_2 = h(t)\]

Looking at picture we see \( z_1 - z_2 = h(t) \)

We have another formula for the headloss

\[h_L = \frac{f L}{D} \frac{v^2}{2g}\] for major losses in the pipe \( \Rightarrow \)

\[h(t) = \frac{f L}{D} \frac{v^2}{2g}\]
Note the speed at which the water surfaces approach each other is $V_1 + V_2$. Thus,

$$\frac{dh}{dt} = -V_1 - V_2 = -2V_1 = -2\frac{\pi D^2}{4A} = -V\frac{\pi D^2}{2A} \implies$$

Recall Cons. of Mass Result

$$V = -\frac{2A}{\pi D^2} \frac{dh}{dt}$$

Plug this into $h(t) = \frac{F}{D} \frac{V^2}{2g} = \frac{F}{D} \frac{1}{2g} \left(-\frac{2A}{\pi D^2} \frac{dh}{dt}\right)^2$

$$= \frac{F}{D} \frac{1}{2g} \frac{4A^2}{\pi^2 D^4} \left(\frac{dh}{dt}\right)^2$$

$$= \frac{2F \frac{L}{gD^5}}{(\frac{A}{\pi})^2} \left(\frac{dh}{dt}\right)^2$$

So our equation for $h(t)$ is an ugly differential equation

$$h(t) = \frac{2F \frac{L}{gD^5}}{(\frac{A}{\pi})^2} \left(\frac{dh}{dt}\right)^2$$

(6) Solve for $h(t)$ if $D = 0.1$ in. Assume and confirm that the flow is laminar.

For laminar flow in pipes $f = \frac{64}{Re} = \frac{64\nu}{vD}$

(Recall $V = -\frac{2A}{\pi D^2} \frac{dh}{dt}$ =)
\[ S = -\frac{64\nu \pi D^2}{D^2} \left( \frac{dh}{dt} \right)^{-1} = -\frac{32\pi \nu D}{A} \left( \frac{dh}{dt} \right)^{-1} \]

Plug this into our ODE for \( h(t) \) from part (a) \[ h(t) = \frac{2L}{g D^2} \left( -\frac{32\pi \nu D}{A} \right) \left( \frac{dh}{dt} \right) \left( \frac{A^2}{\pi^2} \right) \left( \frac{dh}{dt} \right)^2 = -\frac{64\nu LA \cdot dh}{\pi g D^4 \cdot dt} \]

\[ \frac{dh}{dt} = -\frac{\pi g D^4}{64\nu LA} \]

Recall the solution to the first order ODE \[ \frac{dy}{dt} = -\alpha y \] is just \[ y(t) = y(t=0) e^{-\alpha t} \]

Thus

\[ h(t) = h(0) \exp \left( -\frac{\pi g D^4}{64\nu LA} t \right) \]

Confirm that the flow is laminar by checking if \( Re \leq 2100 \)

\[ Re = \frac{VD}{\nu} \] and recall we found \( v = -\frac{2A}{\pi D^2} \frac{dh}{dt} \)

For the laminar case, we just found \( \frac{dh}{dt} = -\frac{\pi g D^4}{64\nu LA} h(t) \)

\[ \Rightarrow v = +\frac{g D^2}{32\nu L} h(t) \]

As \( h(t) \) decreases, \( v \) only gets smaller, so the highest possible \( v \) occurs at \( t=0 \)

\[ V_{\text{max}} = \frac{g D^2 h(0)}{32\nu L} \]
\[ \text{Re}_\text{max} = \frac{V_{\text{max}} D}{\nu} = \frac{g D^2 h(0)}{32 \nu L} = \frac{g D^3 L(0)}{32 \nu^2 L} \]

\[ = \frac{(32.2 \frac{ft}{s})(0.1 \times \frac{1}{12} \text{ft})^3 (2 \text{ft})}{32 (1 \times 10^{-5} \frac{ft^2}{s})^2 (25 \text{ft})} = 4.66 \]

\[ \text{Re}_\text{max} = 4.66 < 2100 \Rightarrow \text{Flow is laminar} \]

(c) How would you solve for \( h(t) \) if \( D = 0.1 \text{ft} \)?

In this case, assuming laminar flow

\[ \text{Re}_\text{max} = \frac{(32.2 \frac{ft}{s})(0.1 \text{ft})^3 (2 \text{ft})}{32 (1 \times 10^{-5} \frac{ft^2}{s})^2 (25 \text{ft})} = 8 \times 10^6 >> 2100 \]

So there is no way the flow is laminar.

For \( \text{Re} > 2000 \) we can use the Swan \\& Jain formula, noting \( h_L = h(t) \)

\[ Q = -0.965 \left( \frac{g D^3 h(t)}{L} \right)^{0.5} \log \left[ \frac{1}{3.7 D} + \left( \frac{3.17 D^2 L}{g D^3 h(t)} \right)^{0.5} \right] \]

If we know \( Q \) we can find \( \frac{dh}{dt} \) because

\[ Q = A v_1 = A \left( -\frac{1}{2} \frac{dh}{dt} \right) \Rightarrow \frac{dh}{dt} = -\frac{2Q}{A} \]

Note we can compute \( \text{Re} \) knowing \( Q \)

\[ \text{Re} = \frac{VD}{\nu} = \frac{4Q}{\pi D^2 \nu} = \frac{4Q}{\pi D \nu} \Rightarrow \text{Re} = \frac{4Q}{\pi D \nu} \]

With these 3 formulae we can follow a simple algorithm to find \( h(t) \)
% Assign known quantities

\[ h_0 = 2 \]
\[ L = 2.5 \]
\[ D = 0.1 \]
\[ A = 10 \]  
\[ \epsilon = 0.005 \]
\[ \mu = 1 \times 10^{-5} \]
\[ g = 32.2 \]

% Compute initial flow rate and Re

\[ Q = \frac{m}{A} \]  (equation A)
\[ Re = \frac{m}{\mu} \]  (equation C)

% Iterate until flow is laminar

\[ i = 1 \]
\[ dt = 1 \]
while \[ Re > \text{2100} \]
\[
\frac{dh}{dt} = -\frac{2Q}{A}
\]
\[ h_i = h_{i-1} + \frac{dh}{dt} \cdot dt \]  (equation B)
\[ Q = \frac{m}{A} \]  (equation A)
\[ Re = \frac{m}{\mu} \]  (equation C)
\[ i = i + 1 \]
end

% Once \[ Re \leq \text{2100} \] use formula for laminar flow

\[ t = dt \]

for \[ j = i+1 \] to \[ N \]
\[ h_j = h_i \cdot \exp \left( -\frac{\pi g D y}{64 \nu L A} \cdot t \right) \]
\[ t = t + dt \]
end