A jet of alcohol strikes the vertical plate in Fig. P3.112. A force $F \approx 425$ N is required to hold the plate stationary. Assuming there are no losses in the nozzle, estimate (a) the mass flow rate of alcohol and (b) the absolute pressure at section 1.

Solution: A momentum analysis of the plate (e.g. Prob. 3.40) will give

$$F = mV_2 = \rho A_2 V_2^2 = 0.79(998)\frac{\pi}{4}(0.02)^2 V_2^2 = 425 \text{ N},$$

solve for $V_2 \approx 41.4 \text{ m/s}$

whence $m = 0.79(998)(\pi/4)(0.02)^2(41.4) \approx 10.3 \text{ kg/s} \quad \text{Ans. (a)}$

We find $V_1$ from mass conservation and then find $p_1$ from Bernoulli with no losses:

Incompressible mass conservation: $V_1 = V_2 (D_2/D_1)^2 = (41.4)\left(\frac{2}{5}\right)^2 \approx 6.63 \text{ m/s}$

Bernoulli, $z_1 = z_2$: $p_1 = p_2 + \frac{1}{2} \rho \left(V_2^2 - V_1^2\right) = 101000 + \frac{0.79(998)}{2}[ (41.4)^2 - (6.63)^2 ]$

$\approx 760,000 \text{ Pa} \quad \text{Ans. (b)}$
A free liquid jet, as in Fig. P3.115, has constant ambient pressure and small losses; hence from Bernoulli’s equation \( z + V^2/(2g) \) is constant along the jet. For the fire nozzle in the figure, what are (a) the minimum and (b) the maximum values of \( \theta \) for which the water jet will clear the corner of the building? For which case will the jet velocity be higher when it strikes the roof of the building?

Solution:  The two extreme cases are when the jet just touches the corner \( A \) of the building. For these two cases, Bernoulli’s equation requires that

\[
V_A^2 + 2gz_A = (100)^2 + 2g(0) = V_A^2 + 2gz_A = V_A^2 + 2(32.2)(50). \quad \text{or} \quad V_A = 82.3 \ \text{ft/s}
\]

The jet moves like a frictionless particle as in elementary particle dynamics:

\[
\begin{align*}
\text{Vertical motion:} & \quad z = (V_1 \sin \theta) t - \frac{1}{2} gt^2; \\
\text{Horizontal motion:} & \quad x = (V_1 \cos \theta) t
\end{align*}
\]

Eliminate “t” between these two and apply the result to point \( A \):

\[
z_A = 50 = x_A \tan \theta - \frac{gx_A^2}{2V_1^2 \cos^2 \theta} = 40 \tan \theta - \frac{(32.2)(40)^2}{2(100)^2 \cos^2 \theta}; \quad \text{clean up and rearrange:}
\]

\[
\tan \theta = 1.25 + 0.0644 \sec^2 \theta, \quad \text{solve for} \quad \theta = 85.94^\circ \quad \text{Ans. (a)} \quad \text{and} \quad 55.40^\circ \quad \text{Ans. (b)}
\]

Path (b) is shown in the figure, where the jet just grazes the corner \( A \) and goes over the top of the roof. Path (a) goes nearly straight up, to \( z = 155 \) ft, then falls down to pt. \( A \). In both cases, the velocity when the jet strikes point \( A \) is the same, 82.3 ft/s.
P3.124  A necked-down section in a pipe flow, called a venturi, develops a low throat pressure which can aspirate fluid upward from a reservoir, as in Fig. P3.124. Using Bernoulli’s equation with no losses, derive an expression for the velocity $V_1$ which is just sufficient to bring reservoir fluid into the throat.

![Diagram of venturi flow](image)

Fig. P3.124

**Solution:** Water will begin to aspirate into the throat when $p_a - p_1 = \rho gh$. Hence:

Volume flow: $V_1 = V_2 \left( \frac{D_2}{D_1} \right)^2$; Bernoulli ($\Delta z = 0$): $p_1 + \frac{1}{2} \rho V_1^2 \approx p_{am} + \frac{1}{2} \rho V_2^2$

Solve for $p_a - p_1 = \rho \left( \alpha^4 - 1 \right) V_2^2 \geq \rho gh$, $\alpha = \frac{D_2}{D_1}$, or: $V_2 \geq \sqrt{\frac{2gh}{\alpha^4 - 1}}$ **Ans.**

Similarly, $V_{1,\text{min}} = \alpha^2 V_{2,\text{min}} = \sqrt{\frac{2gh}{1 - \left(\frac{D_1}{D_2} \right)^4}}$ **Ans.**
P3.127 An open water jet exits from a nozzle into sea-level air, as shown, and strikes a stagnation tube. If the centerline pressure at section (1) is 110 kPa and losses are neglected, estimate (a) the mass flow in kg/s; and (b) the height $H$ of the fluid in the tube.

![Fig. P3.127](image)

**Solution:** Writing Bernoulli and continuity between pipe and jet yields jet velocity:

$$p_1 - p_a = \frac{\rho}{2} V_{jet}^2 \left[ 1 - \left( \frac{D_{jet}}{D_1} \right)^4 \right] = 110000 - 101350 = \frac{998}{2} V_{jet}^2 \left[ 1 - \left( \frac{4}{12} \right)^4 \right].$$

Solve $V_{jet} = 4.19 \text{ m/s}$

*Then the mass flow is* $\boxed{\dot{m} = \rho A_{jet} V_{jet} = 998 \frac{\pi}{4} (0.04)^2 (4.19) = 5.25 \text{ kg/s} \quad \text{Ans. (a)}}$

(b) The water in the stagnation tube will rise above the jet surface by an amount equal to the stagnation pressure head of the jet:

$$H = R_{jet} + \frac{V_{jet}^2}{2g} = 0.02 \text{ m} + \frac{(4.19)^2}{2(9.81)} = 0.02 + 0.89 = 0.91 \text{ m} \quad \text{Ans. (b)}$$
P3.131 In Fig. P3.131 both fluids are at 20°C. If $V_1 = 1.7$ ft/s and losses are neglected, what should the manometer reading $h$ ft be?

Solution: By continuity, establish $V_2$:

$$V_2 = V_1 \left( \frac{D_1}{D_2} \right)^2 = 1.7 \left( \frac{3}{1} \right)^2 = 15.3 \text{ ft/s}$$

Now apply Bernoulli between 1 and 2 to establish the pressure at section 2:

$$p_1 + \frac{\rho}{2} V_1^2 + \rho g z_1 = p_2 + \frac{\rho}{2} V_2^2 + \rho g z_2,$$

Fig. P3.131

or: $p_1 + (1.94/2)(1.7)^2 + 0 \approx 0 + (1.94/2)(15.3)^2 + (62.4)(10)$, $p_1 = 848$ psf

This is gage pressure. Now the manometer reads gage pressure, so

$$p_1 - p_a = 848 \text{ lbf/ft}^2 = (\rho_{\text{mercury}} - \rho_{\text{water}}) gh = (846 - 62.4)h,$$

solve for $h \approx 1.08$ ft. Ans.
P4.19 An incompressible flow field has the cylindrical velocity components \( v_\theta = Cr, v_z = K(R^2 - r^2) \), \( v_r = 0 \), where \( C \) and \( K \) are constants and \( r \leq R, z \leq L \). Does this flow satisfy continuity? What might it represent physically?

**Solution:** We check the incompressible continuity relation in cylindrical coordinates:

\[
\frac{1}{r} \frac{\partial}{\partial r} (rv_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0 = 0 + 0 + 0 \quad \text{satisfied identically} \quad \text{Ans.}
\]

This flow also satisfies (cylindrical) momentum and could represent laminar flow inside a tube of radius \( R \) whose outer wall \((r = R)\) is rotating at uniform angular velocity.
P4.20 A two-dimensional incompressible velocity field has \( u = K(1 - e^{-\alpha y}) \), for \( x \leq L \) and \( 0 \leq y \leq \infty \). What is the most general form of \( v(x, y) \) for which continuity is satisfied and \( v = v_0 \) at \( y = 0 \)? What are the proper dimensions for constants \( K \) and \( \alpha \)?

**Solution:** We can find the appropriate velocity \( v \) from two-dimensional continuity:

\[
\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -\frac{\partial}{\partial x}[K(1 - e^{-\alpha y})] = 0, \quad \text{or:} \quad v = \text{fcn}(x) \text{ only}
\]

Since \( v = v_0 \) at \( y = 0 \) for all \( x \), then it must be that \( v = v_0 = \text{const} \quad \text{Ans.} \)

The dimensions of \( K \) are \( \{K\} = \{L/T\} \) and the dimensions of \( \alpha \) are \( \{L^{-1}\} \). \( \text{Ans.} \)