3.20 Review

- *Angular Momentum Examples* - rotating CV (one arm sprinkler) and fixed CV (pipe flow bracket).

3.21 Incompressible 1-D Flow With No Shaft Work

Rearranging our 1-D head form and setting $w_s = 0$ we have

$$\left( \frac{P}{\gamma} + \frac{v^2}{2g} + z \right)_{\text{out}} = \left( \frac{P}{\gamma} + \frac{v^2}{2g} + z \right)_{\text{in}} - \frac{\hat{u}_{\text{out}} - \hat{u}_{\text{in}} - q}{g}$$

Now, defining $\frac{\hat{u}_{\text{out}} - \hat{u}_{\text{in}} - q}{g} = h_{\text{loss}} = h_f$

where we can think of $h_f$ as the friction losses and we see that $h_f > 0$ (note that in adiabatic flow, say a perfectly insulated pipe, friction will heat the flow, therefore $\hat{u}_{\text{out}} > \hat{u}_{\text{in}}$ and hence $h_f > 0$). Thus we write

$$\left( \frac{P}{\gamma} + \frac{v^2}{2g} + z \right)_{\text{out}} = \left( \frac{P}{\gamma} + \frac{v^2}{2g} + z \right)_{\text{in}} - h_f$$

3.21.1 Example – Gas Pipeline

Consider the following pipe flow:

If $Q = 75$ m$^3$/s, the pipe radius is $r = 6$ cm, the inlet pressure is maintained at 24 atm by a pump, the outlet vents to the atmosphere, the pipe rises 150 m from inlet to outlet
and the pipe length is 10 km, what is $h_f$? What is the velocity head?

$h_f=198$ m Therefore the friction loss is greater than the $\Delta z$ and the pump must drive against both!

The velocity head is only 0.17 m!

Note that the length did not come into our solution. $h_f$ includes the total losses along the pipe due to friction effects and hence includes the effect of length implicitly. We will see more about this a bit later in the course.

### 3.22 Variations From Uniform Flow

As we have discussed we frequently assume that a flow is 1-D while we know in actuality it is not. Often this is an excellent assumption but sometimes the assumption is not as good and we may wish to correct for the effects of the dependence of the velocity on position. The term that is effected in the energy equation is the flux term. If we wish to use the average velocity, $\langle V \rangle$, as representative of a 1-D velocity equivalent to the 2-D velocity then we have

$$\int_{CS} \frac{v^2}{2} \rho (\vec{v} \cdot \vec{n}) \, dA = \dot{m} \left( \frac{\alpha_{out}}{2} \langle V \rangle_{out}^2 - \frac{\alpha_{in}}{2} \langle V \rangle_{in}^2 \right)$$
where $\alpha$ is known as the kinetic constant and it accounts for the effect of the non-uniform velocity profile on the surface flux of energy. The definition of the mean velocity is

$$\langle V \rangle = \frac{\int \rho (\vec{v} \cdot \vec{n}) \, dA}{\rho A}$$

which for incompressible flows with the velocity vector normal to the control surface reduces to simply $\langle V \rangle = Q/A$. Hence

$$\dot{m} \frac{\alpha_{\text{out}}}{2} \langle V \rangle^2 = \int_{CS} \frac{v^2}{2} \rho (\vec{v} \cdot \vec{n}) \, dA$$

and

$$\alpha = \frac{\int_{CS} v^2 \rho (\vec{v} \cdot \vec{n}) \, dA}{\dot{m} \langle V \rangle^2}$$

**Example**

Consider the Laminar flow through a pipe sitting in a uniform velocity field in water:

What is the head loss?

$$h_L = \frac{V^2}{2}$$