3.26 Review

- A special case of the Energy Equation – The Bernoulli Equation

\[ \frac{P}{\gamma} + \frac{v^2}{2g} + z = h_0 \]

Can think of it as the inviscid (frictionless) conservation of energy equation (i.e., \( h_f = 0 = h_s \)).

- Stagnation pressure – the sum of the static pressure and the dynamic pressure.

\[ P_s = P + \rho \frac{v^2}{2} \]

- Pitot-static tube – \( V = \sqrt{\frac{2}{\gamma} \frac{\Delta P}{\rho}} = \sqrt{\frac{2}{\gamma} \frac{\Delta H}{\rho}} = \sqrt{2g \Delta H} \), a particular case of the Bernoulli equation, where \( \Delta P \) or \( \Delta H \) is the measured differential pressure or head, respectively, between the stagnation and static pressures.

3.27 Irrotational Flow and Bernoulli

Consider

\[
\begin{align*}
\frac{P}{\gamma} + \frac{v^2}{2g} + z &= h_0 \\
0 + \frac{v^2}{2g} + 0 &= h_0 = \frac{v^2}{2g} \\
\frac{h_2 \gamma}{\gamma} + \frac{v^2}{2g} - h_2 &= h_0 \quad \checkmark \\
\frac{h_3 \gamma}{\gamma} + \frac{v^2}{2g} - h_3 &= h_0 \quad \checkmark \\
\frac{h \gamma}{\gamma} + 0 - h &= 0 \neq h_0
\end{align*}
\]
Notice that in the unsheared regions (uniform flow) \( h_0 \) = a constant across streamlines while where shear exists (e.g., shear is non-zero), \( h_0 \) varies across the streamlines. More strictly speaking we actually want to know if the flow is rotational. Our test is if we stick a small neutrally buoyant \(+\) shaped probe in the flow and see if it will rotate. In uniform flow it will not, in a linear shear, like the shear profile shown here, it will. Hence we say that \( h_0 \) is constant in irrotational (non rotational) flows. This allows us to connect Bernoulli points that are not on the same streamline in flows that are irrotational, further expanding the power of the Bernoulli equation but also the opportunities to misuse it!

### 3.28 Energy and the Hydraulic Grade Line

As we have seen we can write the head form of the Energy equation as

\[
\frac{P}{\gamma} + \frac{v^2}{2g} + z = H = \text{Energy Grade Line (EGL)}
\]

In the case of Bernoulli flows the energy grade line is simply a constant since by assumption energy is conserved (there is no mechanism to gain/lose energy). For other flows it will drop due to frictional losses or work done on the surroundings (e.g., a turbine) or increase due to work input (e.g., a pump). Note that this is the head that would be measured by a Pitot tube.

We can also write

\[
\frac{P}{\gamma} + z = \text{Hydraulic Grade Line (HGL)}
\]

and we see that the HGL is due to static pressure – the height a column of fluid would rise due to pressure at a given elevation or in other words the head measured by a static pressure tap or the piezometric head.

**Example – Venturi Flow Meter**
Consider

\[ Q = A_2 V_2 = A_2 \left[ \frac{2g\Delta h}{1 - \left( \frac{A_2}{A_1} \right)^2} \right]^\frac{1}{2} \]

### 3.28.1 Frictional Effects

If we have abrupt losses, say at a contraction, a simple way of accounting for this is through a discharge coefficient. We can write a modified form of Torricelli’s formula for incompressible flow

\[ \langle V \rangle = \frac{Q}{A} = C_d \sqrt{2gh} \]

where \( C_d \) is the discharge coefficient and is 1 for frictionless (inviscid) flow and can range down to about 0.6 for flows strongly effected by friction. Note we can handle non-uniform (violation of 1-D assumption) flow effects with a \( C_d \) as well.

### 3.28.2 Vena Contracta Effect

For a flow to get around a sharp corner there would need to be an infinite pressure gradient, which of course does not happen. Hence if the boundary changes directions too rapidly at an exit, the flow separates from the exit and forms what is known as a \textit{vena}
Clearly $A_j/A \leq 1$. For a round sharp-cornered exit the coefficient is $C_c = A_j/A = 0.61$ and typical values of the coefficient fall in the range $0.5 \leq C_c \leq 1.0$.

### 3.29 Cavitation and the Bernoulli Equation

Consider the following flow geometry:

If we assume over such a short section friction is negligible then, given the constant $z$, the Bernoulli equation reduces to

$$P_0 = P + \rho \frac{v^2}{2}$$

Therefore high pressure occurs when the velocity is low and as the velocity increases the pressure drops. Plotting the variation of the dynamic and static pressure:

we find that the pressure might fall below the vapor pressure of the fluid and hence cavitate. Thus in general we should be concerned about cavitation any time we are dealing with relatively high velocities.