7.11 Review

*Momentum integral equation*

\[ \tau_w = \rho U_\infty^2 \frac{d\theta}{dx} \]

*Von Kármán assumed*

\[ \frac{u(x, y)}{U_\infty} = 2y = \frac{y^2}{\delta} - \frac{y^2}{\delta^2} \]

and found

\[ \frac{\delta}{x} = \frac{5.5}{Re_{x}^{0.5}} \Rightarrow \delta = \frac{5.5}{Re_{x}^{0.5}} \cdot x \propto x^{1/2} \]

*Skin Friction Coefficient* \( c_f = \frac{\tau_w}{\frac{1}{2} \rho U^2} \Rightarrow c_f = \frac{0.73}{Re_{x}^{1/2}} \propto x^{-1/2} \)

*Boundary layer equations* – an excellent approximation to the Navier-Stokes in BL flows if you are not too close to the \( x = 0 \) point.

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \]

*Blasius’ Solution* the “exact” solution for laminar BLs \( \frac{\delta}{x} = \frac{5.0}{Re_{x}^{1/2}} \) and \( c_f = \frac{0.664}{Re_{x}^{1/2}} \)

within 10% of Von Kármán’s solution, may use either for CEE 3310.

7.12 Turbulent Boundary Layer Growth Rate

Recall

\[ c_f = \frac{2\tau_w}{\rho U^2} \]

and

\[ \tau_w = \rho U^2 \frac{d\theta}{dx} \]

Substituting

\[ C_f = 2 \frac{d\theta}{dx} \]
Following a suggestion of Prandtl and relating this to the log-law (see textbook starting around equation 7.34) one can use the 1/7th power law form of the turbulent boundary layer to find that $\Theta = \frac{7}{72}\delta$ and after some further manipulation one arrives at:

\[ \frac{\delta}{x} = \frac{0.16}{\text{Re}_x^{1/7}} \text{ hence } \delta \propto x^{6/7} \]

and

\[ c_f = \frac{0.027}{\text{Re}_x^{1/7}} \text{ hence } c_f \propto x^{-1/7} \]

### 7.13 Drag & Lift

When a body moves through a fluid (or a fluid flows past a body) forces occur at the body-fluid interface.

- Tangential (or shear) stresses ⇒ $\tau_w$
- Normal stresses ⇒ $P$.

$\tau_w$ and $P$ are functions of position. If we integrate the forces over the body’s surface we call the resultants:

- **Drag** ⇒ Resultant force in direction of upstream flow ($\vec{U}_\infty$)
- **Lift** ⇒ Resultant force in direction normal to $\vec{U}_\infty$.

Considering that the forces are a result of the stresses acting on areas that may have components normal and perpendicular to $\vec{U}_\infty$, drag & lift may be a result of both net
pressure & shear induced forces. Drag

If we know $P(x, y, z)$ over whole body we can integrate and find the pressure drag also commonly referred to as the form drag. Similarly if we know $\tau_w(x, y, z)$ we can integrate and find the friction drag.

In general we do not know either of these functions. Measurement of pressure is usually possible but to measure the shear stress distribution is challenging!

In general we rely on experiments to determine the total drag on an object (the sum of the pressure and friction drags). We express the drag ($D$) nondimensionally as a drag coefficient, $C_D$

$$C_D = \frac{D}{\frac{1}{2} \rho V^2 A}$$

where $A$ is an appropriately chosen area with respect to the stresses generating the drag.

**Characteristic Area**

There are three standard area types used in drag coefficients:

1. **Frontal Area** – the area seen by the flow stream. For drag coefficients dominated by pressure drag (e.g., blunt objects such as cylinders and spheres) this is often the appropriate choice of area.
2. **Planform Area** – area as seen from above (e.g., perpendicular to flow stream). For drag coefficients dominated by friction drag (e.g., wide flat bodies such as wings and hydrofoils) this is often the appropriate choice of area.
3. **Wetted Area** – area of wetted surface. Often used for boats and other surface water vessels where friction drag is dominant.
7.14 Drag Coefficient Dependencies – Geometry, Re & Roughness

In general $C_D = \phi(\text{Re}, \text{Fr}, \epsilon/L, \text{Ma}, \text{Geometry})$.

Geometry

Narrow wake = low pressure (form) drag!

Laminar vs Turbulent Boundary Layer

Turbulent boundary layers are more robust to adverse pressure gradients. This means that a turbulent boundary layer will tend to follow the curvature of bodies longer than a laminar boundary layer. Hence given the same geometry the boundary layer will stay attached to the body longer for turbulent flows and in general produce a narrower wake. As just seen narrower wakes lead to lower pressure drag and frequently lower total drag. The price paid for turbulence is higher friction drag but if pressure drag is the dominant drag component this may be well worth the price!

As an example look at the bowling balls entering water (taken from White (1999), his figure 7.14)

Designers often take advantage of tripping a boundary layer turbulent. Perhaps the best known example is the golf ball dimpling. Consider the following data for $C_D$ for a sphere
as a function of roughness and Re (Text backhand Fig 9.25):

![Fig. 7.14 Strong differences in laminar and turbulent separation on an 8.5-in bowling ball entering water at 25 ft/s: (a) smooth ball, laminar boundary layer; (b) same entry, turbulent flow induced by patch of nose-sand roughness. (U.S. Navy photograph, Ordnance Test Station, Pasadena Annex.)](image)

![Fig. 9.25 The effect of surface roughness on the drag coefficient of a sphere in the Reynolds number range for which the laminar boundary layer becomes turbulent (Ref. 5).](image)

The drag coefficient for a well-hit golf ball (defined as a ball with Re > $4 \times 10^4$) is considerably less than would be for a smooth ball at the same Re.
7.15 Example – Terminal Velocity of a Parachuter

Consider a parachute with diameter 4 m carrying a person and gear such that the total mass is 80 kg. If the parachuter jumps from high enough that terminal velocity (e.g., steady state - no acceleration) is achieved, what is this velocity?

\[ U = 8.6 \text{ m/s} \]

7.16 Biological Drag Reduction

Organisms adapt to survive high winds and currents (See many publications by Mark Denny for coastal organisms or Steven Vogel).

Trees are an excellent example. Their flexible structure allows them to reconfigure to reduce their drag as wind speed increases. Consider the following measured drag coefficient data for pine and spruce trees (Johnson, R. C.; Ramey, G. E.; O’Hagen, D.S. (1983). Wind induced forces on trees. *J. Fluids Eng.* **104**, 25-30.):

<table>
<thead>
<tr>
<th>( U_\infty (m/s) )</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_D )</td>
<td>1.2±0.2</td>
<td>1.0±0.2</td>
<td>0.7±0.2</td>
<td>0.5±0.2</td>
</tr>
</tbody>
</table>

Vogel (Vogel, S (1989). Drag and reconfiguration of broad leaves in high winds. *J.*
Exp. Bot. 40, 941-948.) found that tulip tree leaves curl to reduce drag as wind speed increases.

Denny et al. (Denny, M. W.; Gaylord, B.P.; Cowen, E.A. (1997). Flow and flexibility II: The roles of size and shape in determining wave forces on the bull kelp, Nereocystis luetkeana. J. Exp. Biology 200, 3165-3183.) found that kelp reduces its drag by “going with the flow”. This reduces the relative velocity (drag force \( \propto U^2 \)) and hence the drag. This leads to potential large inertial forces as the kelp comes to the end of its stipe (dog at the end of its leash!) but nature has worked this out too. An interesting feature in kelp is that it does not grow isometrically as its juvenile shape would lead to considerable drag at mature scale. Hence it adapts its shape with age to minimize drag.