3.28 Review

- A special case of the Energy Equation – The Bernoulli Equation: Applicable if you are on a streamline, at steady state, the flow can be well approximated as frictionless and no work or heating is done along the streamline

\[ \frac{P}{\gamma} + \frac{v^2}{2g} + z = h_0 \]

Can think of it as the inviscid (frictionless) conservation of energy equation (i.e., \( h_f = 0 = h_s \)).

- Stagnation pressure – the sum of the static pressure and the dynamic pressure.

\[ P_s = P + \rho \frac{v^2}{2} \]

- Pitot-static tube – \( V = \sqrt{2\frac{\Delta P}{\rho}} = \sqrt{2\frac{\gamma \Delta H}{\rho}} = \sqrt{2g \Delta H} \), a particular case of the Bernoulli equation, where \( \Delta P \) or \( \Delta H \) is the measured differential pressure or head, respectively, between the stagnation and static pressures.

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3.29 Irrotational Flow and Bernoulli

Consider

\[
\frac{P}{\gamma} + \frac{v^2}{2g} + z = h_0
\]

\[
0 + \frac{v^2}{2g} + 0 = h_0 = \frac{v^2}{2g}
\]

\[
\frac{h_2\gamma + v^2}{2g} - h_2 = h_0 \quad \checkmark
\]

\[
\frac{h_3\gamma + v^2}{2g} - h_3 = h_0 \quad \checkmark
\]

\[
\frac{h\gamma}{\gamma} + 0 - h = 0 \neq h_0
\]

Notice that in the unsheared regions (uniform flow) \( h_0 \) = a constant across streamlines while where shear exists (e.g., shear is non-zero), \( h_0 \) varies across the streamlines. More strictly speaking we actually want to know if the flow is rotational. Our test is if we stick a small neutrally buoyant + shaped probe in the flow and see if it will rotate. In uniform flow it will not, in a linear shear, like the shear profile shown here, it will. Hence we say that \( h_0 \) is constant in irrotational (non rotational) flows. This allows us to connect Bernoulli points that are not on the same streamline in flows that are irrotational, further expanding the power of the Bernoulli equation but also the opportunities to misuse it!

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3.30 Cavitation and the Bernoulli Equation

Consider the following flow geometry:

If we assume over such a short section friction is negligible then, given the constant $z$, the Bernoulli equation reduces to

$$P_0 = P + \frac{\rho v^2}{2}$$

Therefore high pressure occurs when the velocity is low and as the velocity increases the pressure drops. Plotting the variation of the dynamic and static pressure:

we find that the pressure might fall below the vapor pressure of the fluid and hence cavitate. Thus in general we should be concerned about cavitation any time we are dealing with relatively high velocities.