1.8 Review

- Dimensional consistency ⇒ \( F = ma \) \([\text{MLT}^{-2}] = [\text{M}][\text{LT}^{-2}]\). A dimensionally consistent equation may be based on physics, an inconsistent equation is certainly not physically based.

- System of units ⇒ B.G., S.I. B

- Thermodynamic properties ⇒ \( \Theta, P, \rho(\Theta, P) \)

- Perfect Gas Law ⇒ make sure you review it!

- Viscosity ⇒ stress \( \propto \) strain rate \( \rightarrow \tau = \mu \frac{du}{dz} \)

1.9 Vapor Pressure

If initially we start with a vacuum, over time a pressure will form as the result of molecular action. Particles leave the surface. Eventually an equilibrium pressure is achieved as the same number of particles leave the surface as return to it. This pressure is known as the vapor pressure of the fluid and is denoted \( p_v \).

As we will see in a few weeks, fluid motions can lead to very low pressures. If \( p \leq p_v \) the fluid will boil. This process is known as cavitation.
1.10 Surface Tension

The water molecule is polar. The $\text{O}^-$ attracts the $\text{H}^+$. Within the fluid this attraction is in balance, i.e., the net force due to all of the polar pairs is zero. However, at the surface half of this force is missing and the surface is pulled toward the fluid interior with a certain energy.

$$\text{surface energy} = \frac{J}{m^2} = \frac{Nm}{m^2} = \frac{N}{m} = \frac{\text{force}}{\text{length}} = \text{tension}$$

hence we refer to this energy as the *surface tension* ($\Upsilon$).

1.10.1 Example – the pressure in a bubble

Tension force = $2\pi R \Upsilon$

Pressure force = $(P_I - P_E)\pi R^2 \Rightarrow \Delta P = P_I - P_E = \frac{2\pi \Upsilon R}{\pi R^2} = \frac{2\Upsilon}{R}$

1.10.2 The Contact Angle

In the case of a bubble we only had to concern ourselves with a liquid–gas interface but often we find we have three phases present (a liquid-gas-solid interface) for example, when you fill your glass with water and you get a contact line around the circumference of the glass at the air-water-glass interface. You’ve all likely noticed that the contact
line rises locally, appearing to adhere to and be lifted by the glass boundary forming what is known as a \textit{meniscus}, the region local to the solid boundary where the gas-liquid interface is curved.

The angle that is formed at this three-phase interface is known as the \textit{contact angle} and is defined as the angle between the line originating from the three-phase contact point tangent to the liquid-gas interface and the tangent to the solid boundary as measured through the liquid, e.g.,

When the liquid seems to spread easily over the boundary, the contact angle is $\theta_c < 90^\circ$ and we refer to the liquid as \textit{wetting}, as in the case of water on glass, which is totally wetting giving $\theta_c \approx 0$. When the liquid resists spreading over the boundary, instead trying to form a droplet, the contact angle is $\theta_c > 90^\circ$ and we refer to the liquid as \textit{non-wetting}, as in the case of water on teflon ($\theta_c \approx 110$).

\textbf{1.10.3 2\textsuperscript{nd} Example – A water barometer}

You are planning on constructing your own water barometer. This will be constructed by filling a long cylindrical glass tube sealed at one end with water and then carefully inverting it so that the mouth of the tube stays wet. The free surface of the water drops to a given elevation and you can measure the height of the water above the reservoir below to calculate the atmospheric pressure. There are at least two important fluid properties that affect the accuracy of your water barometer, what are they?
What minimum diameter must the tube be if you want the capillary induced rise in the tube to be less than 1 mm (assume 20°C water)?

If the atmospheric pressure is 30 in Hg, what is the correction you need to apply to the barometer reading to account for the effect of vapor pressure on your reading?