8.5 Review

Drag & Lift \Rightarrow Laminar vs Turbulent Boundary Layer \Rightarrow Turbulent boundary layers stay attached to bodies longer \Rightarrow Narrower wake! \Rightarrow Lower pressure drag!

Biological drag reduction \Rightarrow plants deform with flow and lower their drag coefficient.

8.6 Uniform Flow

In uniform flow $y_1 = y_2$, $V_1 = V_2 = V$, therefore the energy equation becomes:

$$z_1 - z_2 = S_0 L = h_f$$

Flow is essentially fully developed, therefore we can apply Darcy-Weissbach relations.

$$h_f = f \frac{L}{D_h} \frac{V_{avg}^2}{2g}$$

In open channel flow it is more common to work with the:

$$\text{hydraulic radius} = \frac{A}{P} = \frac{D_h}{4} = R_h$$

where again $(P)$ is the length of the wetted perimeter. Combining the above two equations we have:

$$S_0 L = f \frac{L}{D_h} \frac{V_{avg}^2}{2g} \Rightarrow V = \left(\frac{8g}{f}\right)^{1/2} (R_hS_0)^{1/2}$$

8.7 Chézy Formulas

Chézy defined the coefficient

$$C = \left(\frac{8g}{f}\right)^{1/2}$$

(now called the Chézy coefficient) and found that it varies by a factor of 3. Therefore

$$V = C (R_hS_0)^{1/2}$$

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and

\[ Q = CA (R_h S_0)^{1/2} \]

Manning did field tests and found

\[ C = \left( \frac{8g}{f} \right)^{1/2} \approx \alpha \frac{R_h^{1/6}}{n} \]

where \( n \) is known as Manning’s \( n \) and is a roughness coefficient and \( \alpha \) is a dimensional constant that varies with systems of units (this is not a homogeneous equation, remember?!). For SI \( \alpha = 1 \), for BG \( \alpha = 1.486 \). It is left as an exercise for the student to find the units and verify the conversion.

### 8.8 Manning’s Equation

Substituting Manning’s result into the Chézy formulas we have the celebrated Manning’s equation:

\[ V \approx \frac{\alpha}{n} R_h^{2/3} S_0^{1/2} \quad \text{and} \quad Q \approx \frac{\alpha}{n} A R_h^{2/3} S_0^{1/2} \]

\( n \) varies by a factor of 15 and is tabulated in your text and more extensively elsewhere.

### 8.9 Example – Fall Creek Flow

Let’s consider Fall Creek where the USGS operates a gaging station. Looking over yesterday’s record we see a local maximum flow depth occurred on 11/18/2014 @ 06:00 EST of \( d = 1.85' \). If we estimate \( S_0 \approx 0.001, \ n = 0.035 \) from table 10.1 in text, assuming somewhere between clean and straight and sluggish with deep pools and a width of \( b = 50', d = 1.85' \) we can ask the question, what is \( Q \)?

\[ Q = \frac{\alpha}{n} A R_h^{2/3} S_0^{1/2} \]

\[ R_h = \frac{A}{P} = \frac{bd}{2d + b} \approx d \quad \text{if} \quad b \gg d \]

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therefore \[ Q = \frac{\alpha \cdot b d^{5/3} S_0^{1/2}}{n} = \frac{1.486 \text{ ft}^{1/3}/s}{0.035} \times (50 \text{ ft})(1.85 \text{ ft})^{5/3} \sqrt{0.001} \]

\[ \Rightarrow \quad Q = 484 \text{ CFS} \]

Actual value from calibrated flow gage was 335 CFS. Just using ball park estimates we were within 25% – pretty good! To be more accurate we would need a survey of the river slope and the wetted perimeter. We could get these more accurately from a USGS topographic map but to be truly accurate we would send a survey team out to measure directly in the field.

**Second example - Given** \( Q \), **find** \( d \)

If \( d \ll b \) straightforward – the equation is explicit and we just solve for \( d \) directly by substitution into Manning’s equation.

However, let’s not make this assumption and let’s consider a trapezoidal shaped channel with the geometry shown below and \( Q = 484 \text{ CFS} \), \( S_0 = 0.001 \), and \( n = 0.035 \):

\[
A = \left(\frac{20 + b}{2}\right) d
\]

Using Pythagoras’ theorem we can write

\[
\mathcal{P} = 20 + 2 \sqrt{\left(\frac{b - 20}{2}\right)^2 + d^2}
\]

and

\[
b = 20 + 2d \frac{15}{1.1}
\]
Therefore

\[ A = \left(20 + d^{15}_{1.1}\right) d \]

\[ P = 20 + 2 \sqrt{\left(d^{15}_{1.1}\right)^2 + d^2} \]

Therefore:

\[ Q = \frac{\alpha A^{5/3}}{n \frac{P^{2/3} S_0^{1/2}}{0.035}} \frac{\left[\left(20 + d^{15}_{1.1}\right) d\right]^{5/3}}{\left[20 + 2 \sqrt{\left(d^{15}_{1.1}\right)^2 + d^2}\right]^{2/3}} \sqrt{0.001} = 282 \text{ ft}^3/\text{s} \]

Ouch - what to do? We can solve this iteratively by guessing a \( d \) and then fiddling until we find \( Q = 484 \) CFS. Alternatively, we can use an equation solver to find the result. I used the function \( \text{fzero} \) in Matlab which is a nonlinear root finder and found the zeroes to the expression:

\[ \text{residual} = Q - \frac{\alpha A^{5/3}}{n \frac{P^{2/3} S_0^{1/2}}{0.035}} \]

with an initial guess of \( d = 3 \) ft. The result is \( d = 3.41 \) ft.