8.14 Review

\[ V = \frac{\alpha}{n} R_{h}^{2/3} S_{0}^{1/2} \] and for \( b \gg y, \) \( R_{h} \approx y \Rightarrow V \approx \frac{\alpha}{n} y^{2/3} S_{0}^{1/2} \) and \( Q = VA \approx \frac{\alpha}{n} b y^{5/3} S_{0}^{1/2} \)

Note - the above solved for depth \((y)\) defines what is known as the normal flow depth \((y_{N})\) - that is the flow that results under uniform flow conditions, when the free surface slope is equal to the bed slope is equal to the slope of the energy grade line, due to a friction gravity balance. It may be super or sub critical.

8.15 Critical and Choked Flows

Looking at the previous two examples if the flow is in either of the two states (c) we see that the flow over the sill or through the throat (narrowest part of the channel) is critical. If the sill is raised at all or the throat is narrowed further (e.g., \( q_{2} \) is raised further) the flow depth upstream \((y_{1})\) must increase in order to pass the flow. This is known as a choked flow. Note that the flow depth at the sill (throat) is still \( y_{2} = y_{c} \). Hence the flow is controlled at the sill (throat) and both the upstream and downstream flows are controlled by the sill (throat). This is possible as the flow upstream is subcritical hence information can propagate upstream from the control point to set the water depth upstream \((y_{1})\) and the flow is supercritical downstream hence information can propagate downstream from the control point to set the downstream water depth \((y_{3})\).

In fact the above explains why a fourth possible solution to the two example problems does not exist. Note we did not admit any solutions of the form \( Fr_{1} > 1 \) and \( Fr_{3} < 1 \)
which would look like:

As this would require that a supercritical flow be controlled from downstream and a subcritical flow be controlled from upstream which is not possible.

How do supercritical flows transition to subcritical flows downstream? The hydraulic jump!

8.16 Rapidly Varied Flow

8.16.1 The Hydraulic Jump

As described a transition from super- to sub-critical flow.

- Extremely efficient energy dissipater

- Jump characteristics determined primarily by Fr

If we start with the conservation of energy and the continuity equation (our usual starting point so far) applied to a rectangular open-channel flow of width \( b \) with a hydraulic jump

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we have:

Continuity \[ Q_1 = y_1 b V_1 = y_2 b V_2 = Q_2 \Rightarrow q_1 = q_2 = y_2 V_2 = y_1 V_1 \]

Energy conservation \[ E_1 = E_2 + h_L \Rightarrow y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + h_L \]

where we assume that the jump occurs over a short enough distance that \( h_L \) only includes energy dissipated by the flow in the jump and not by shear stresses at the boundaries (the reasons will become clear in a moment).

- knowns – \( y_1, V_1, g \)
- unknowns – \( y_2, V_2, h_L \)

Therefore we have two equations and three unknowns – we need another equation \( \Rightarrow \) conservation of linear momentum. Our picture:

\[
\gamma \frac{y_1}{2} y_1 b - \gamma \frac{y_2}{2} y_2 b = \dot{m}_{out} - \dot{m}_{in} = \rho Q (V_2 - V_1) = \rho V_1 y_1 b (V_2 - V_1)
\]

Rearranging we have

\[
\frac{y_1^2}{2} - \frac{y_2^2}{2} = \frac{V_1 y_1}{g} (V_2 - V_1)
\]

Combing the above with continuity to eliminate \( V_2 \) and solving for the ratio \( \frac{y_2}{y_1} \) we arrive at

\[
\left( \frac{y_2}{y_1} \right)^2 + \frac{y_2}{y_1} - 2 \frac{V_1^2}{gy_1} = \left( \frac{y_2}{y_1} \right)^2 + \frac{y_2}{y_1} - 2Fr_1^2
\]

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Solving this quadratic equation we have

\[
\frac{y_2}{y_1} = \frac{-1 \pm \sqrt{1 + 8Fr_1^2}}{2} \Rightarrow \frac{y_2}{y_1} = \frac{-1 + \sqrt{1 + 8Fr_1^2}}{2}
\]

since clearly \( \frac{y_2}{y_1} > 0 \) and \( Fr > 1 \).

This result can be combined with the energy and continuity equations to solve for the head loss \( (h_L) \) through the jump.

What does a hydraulic jump look like on a specific energy diagram?