8.17 Review

- **Specific Energy** – a local energy – energy relative to the bed elevation:

  \[ E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gb^2y^2} = y + \frac{q^2}{2gy^2} \]

  where \( q = \frac{Q}{b} = V y \)

- **The Specific Energy Curve** - solution to the cubic:

  At \( Fr = 1 \) \( y = y_c, \ E = E_{\text{min}} = \frac{3}{2} y_c \) where \( y_c = \left( \frac{q^2}{g} \right)^{1/3} = \left( \frac{Q^2}{gb^2} \right)^{1/3} \)

- **Flow over a sill**

- **Flow through a contraction**

8.18 Rapidly Varied Flows – Weirs

8.18.1 Broad-Crested Weir

Consider the following flow:

This is known as the broad-crested weir which is characterized by:

- Sufficiently short that energy loss due to channel friction is negligible \( \Rightarrow \ h_L = 0 \) \( \Rightarrow \) Bernoulli’s equation.

- Sufficiently long horizontal section that hydrostatic flow is a reasonable approximation \( \Rightarrow \) pressure over horizontal section is hydrostatic.
Therefore our starting point is the Bernoulli equation

\[ E_1 = E_2 + h_w \quad \Rightarrow \quad \frac{V_1^2}{2g} + H + h_w = \frac{V_2^2}{2g} + h_w + y_2 \]

Aha! Since the flow upstream is subcritical and there is a region where \( \frac{dy}{dx} \sim 1 \) just upstream of the weir the flow over the horizontal section must be a control and hence \( y_2 = y_c \) and \( V_2^2 = V_c^2 = gy_c \). Therefore we have

\[ \frac{V_1^2}{2g} + H = \frac{y_c}{2} + y_c = \frac{3}{2} y_c \]

Solving for \( y_c \) we have

\[ y_c = \frac{V_1^2}{3g} + \frac{2}{3} H \approx \frac{2}{3} H \quad \text{if} \quad \frac{V_1^2}{2g} \ll H \]

Therefore a reasonable approximation of the flow rate over a weir is

\[ Q = b y_c V_c = b y_c \sqrt{g y_c} = b \sqrt{g y^3/2} = b \sqrt{g y^3/2} = \left(\frac{2}{3}\right)^{3/2} b \sqrt{g H^3} \]

Now the reality is while the above is reasonable, there are energy losses and often broad-crested weirs are used as flow discharge measurement devices. Hence experimental calibration is often preferred and a \textit{weir discharge coefficient} – \( C_d \) is experimentally determined. I.e.,

\[ Q = C_d b \sqrt{g H^3} \]

The literature is full of different formulations such as equations 10.57 and 10.58 in your textbook which yields a maximal \( C_d \) of about 0.54.

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8.18.2 Sharp-Crested Weir

Now our picture looks like:

This is known as the sharp-crested weir. We can get a first-order solution by assuming that the velocity field upstream of the weir is uniform and that the fluid flows horizontally over the weir and that the flow in the overflow section (known as the nappe) is a free jet and hence the pressure is atmospheric. Now our picture is:

Assuming that the energy losses are minimal and following our pictured streamline we have from the Bernoulli equation

\[
\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{u_2^2}{2g} + (h_w + y_2 - h)
\]

where \(u_2\) is the velocity at the streamline elevation over the weir and \(h\) is the distance from the free surface to the elevation of our streamline.

Now, since energy is conserved we know

\[
\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{V_2^2}{2g} + (h_w + y_2)
\]

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and we can write:

\[ u_2 = \left[ 2g \left( h + \frac{V_1^2}{2g} \right) \right]^{\frac{1}{2}} \]

Assuming a rectangular weir of width \( b \) we can find the flow rate by integrating:

\[ Q = \sqrt{2gh} \int_0^{y_2} \left( h + \frac{V_1^2}{2g} \right)^{\frac{1}{2}} dh \]

Now, if \( h_w \gg y_2 \) (which is often the case) the upstream energy is negligible and we can ignore the \( V_1^2 \) term which simplifies the result of our integral too:

\[ Q = \frac{2}{3} \sqrt{2gby_2^{\frac{3}{2}}} \]

very similar to our broad-crested weir result! Analogously we can write a weir coefficient to handle energy losses and geometry effects

\[ Q = C_{wr} \frac{2}{3} \sqrt{2gby_2^{\frac{3}{2}}} \]

The book actually uses \( y_2 \approx \left( \frac{2}{3} H \right) \) and gets

\[ Q = C_d b \sqrt{gH^3} \]

Identical to the broad-crested weir except that the coefficient must differ. They give the coefficient in equation 10.56.

### 8.19 An Example Problem