2.9 Review

- If the surface $S$ is horizontal, $P = a$ constant $\Rightarrow F_R = PA$

- Hydrostatic force on an inclined plane surface

$$F_R = P_C A$$

$$x_R = \frac{I_{xyC} \sin \theta}{P_C A}$$

$$y_R = \frac{I_{xc} \sin \theta}{P_C A}$$

where the subscript $C$ indicates centroid, that is, $P_C$ is the pressure at the centroid of the area of interest, $A$ and the coordinates $x_r, y_r$ are the distances horizontally and down from the centroid, respectively. If, and only if, the fluid column above $A$ is a liquid with a free surface then $P_C = \gamma h_c$, where $h_c$ is the depth of liquid over the centroid. However, if gasses or pressurized headspaces are involved then they must be included when determining $P_C$ (i.e., $P_C = \gamma h_c + P_0$ where $P_0$ is the headspace pressure).

2.10 Hydrostatic Force on a Curved Surface

There are two approaches:

1. $dF_R = P \, dA \Rightarrow$ non-planar hence the integration is hard!

2. Apply a static control volume

$$\sum F_x = 0 = F_{AC} - F_H \Rightarrow F_{AC} = F_H$$

$$\sum F_z = 0 = -F_{AB} - W + F_V \Rightarrow F_V = F_{AB} + Q$$
Where is the center of pressure?

\( F_H \) must be collinear (no shear) with \( F_{AC} \)

\( F_V \) must be collinear with the resultant of \( F_{AB} + W \)

Example – Oil Tanker

\[
F_H = F_{AC} = P_C A = \gamma h C A = \gamma \left( d - \frac{r}{2} \right) rb
\]

Now assume we are interested in solving for a unit width (i.e., \( b = 1 \) m). Therefore

\[
F_H = 10 \frac{kN}{m^3} \left( 24m - \frac{1.5m}{2} \right)(1.5m)(1m) = 349kN
\]

2.10.1 Line of Action

How do we determine the line of action for the resultant force on a curved surface? We could use our moment of inertia based method reviewed above but let’s use pressure prisms here.

\[
F_1 = \gamma (d - r)rb = 10 \cdot 22.5 \cdot 1.5 \cdot 1 = 337.5kN
\]

\[
y_1 = d - \frac{r}{2} = 23.25m
\]

\[
F_2 = \gamma \frac{r}{2} rb = 10 \cdot \frac{1.5}{2} \cdot 1.5 \cdot 1 = 11.25kN
\]

\[
y_2 = d - \frac{r}{3} = 23.5m
\]

Therefore

\[
y_{AC} F_{AC} = F_1 y_1 + F_2 y_2 \quad \Rightarrow \quad y_{AC} = \frac{337.5kN \cdot 23.25m + 11.25kN \cdot 23.5m}{348.75kN} = 23.26m
\]

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Similarly

$$ F_V = 355.3 \text{kN} \quad \text{and} \quad x_{BC} = -0.756 \text{m} \quad (x = 0 \text{ at the tanker’s vertical surface}) $$