6.15 Review

3 Types of Pipe Flow – use Moody diagram (iterations), Haaland (iterations), or Swamee-Jain equations (no iterations!):

I. Find $h_L \Rightarrow \Delta P$

II. Find $V$ (or $Q$)

III. Find $D$

6.15.1 Entrance/Exit Effects

The actual pipe entrance and exit conditions induce a loss which is generally captured as a minor loss:

6.15.2 A second example with Minor Losses

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6.16 Non-circular Pipes

Laminar flow theory can be extended from the Navier-Stokes equations and exact solutions found for fully developed pipe flow in pipes with arbitrary cross-sectional shape. We would find

\[ f = \frac{C}{\text{Re}_{D_H}} \]

where \( C = 64 \pm 25 \) or \( C = 64 \pm 40\% \) and \( \text{Re}_{D_H} \) is the Reynolds number based on the hydraulic diameter \( D_H \) where \( D_H \) is defined as

\[ D_H = \frac{4 \times \text{cross-sectional area}}{\text{Perimeter of cross section}} \]

Note that by definition \( D_H = D \) for a circular pipe.

Turbulent flow

We expect \( f = f(\text{Re}_{D_H}, \epsilon/\text{Re}_{D_H}) \). In fact it has been shown that if the Reynolds number and the non-dimensional roughness are based on \( D_H \) the Moody diagram yields results that are accurate to within 15\% – well within the average uncertainty in pipe roughness values. Hence our approach for turbulent non-circular pipes is simply to use the Moody diagram (or the Colebrook or Haaland or Swamee-Jain formulas) with \( \text{Re}_D \) replaced by \( \text{Re}_{D_H} \).
Flow Meters - Bernoulli Obstruction Theory

Consider the following pipe flow:

If we assume the flow is inviscid then from Bernoulli we can write:

\[ P_1 + \frac{1}{2} \rho V_1^2 = P_2 + \frac{1}{2} \rho V_2^2 \]

From conservation of mass we can write

\[ V_1 = \left( \frac{D_2}{D} \right)^2 V_2 \]

and hence, solving for \( V_2 \):

\[ V_2 \approx \sqrt{\frac{2(P_1 - P_2)}{\rho \left( 1 - \left( \frac{D_2}{D} \right)^4 \right)}} \]

where we use \( \approx \) because we recognize that there is head loss and the diameter of the flow after the vena contracta might be hard to determine. We can turn the \( \approx \) into an \( = \) by multiplying by a calibration coefficient which we then solve for by calibration. We can eliminate the vena contracta concerns by setting the ratio of the diameters to be based on the geometry \( \beta = d/D \) and calibrating for the vena contracta effect as well. Hence we have

\[ Q = A_t V_t = C_d A_t \sqrt{\frac{2(P_1 - P_2)}{\rho \left( 1 - \beta^4 \right)}} \]

where \( A_t = \pi d^2 / 4 \). We expect that \( C_d(\beta, \text{Re}_D) \) hence calibration of the meter results in curves like: