

1.4 Review

- A fluid is a substance that *can not* support a shear stress.
- Liquids differ from gasses in that liquids that do not completely fill a container will form a *free surface* in a gravitational field (and mix minimally with any atmosphere) while a gas will form an atmosphere (and eventually mix with an existing atmosphere).
 ⇒ While buoyancy forces are important in each, gravity is generally an important forcing term in free surface liquid flows and not in atmospheric (gas) flows.
- We consider a fluid to be a *continuum* – i.e., it is continuously differentiable.

1.5 Thermodynamic Properties

- **Temperature** Measure of internal energy level.
- **Pressure** Measure of compressive (normal) stress at a point.

$$P = -\frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) = \frac{F}{A}$$

It is created by the bombardment of the surface by molecules of fluid.

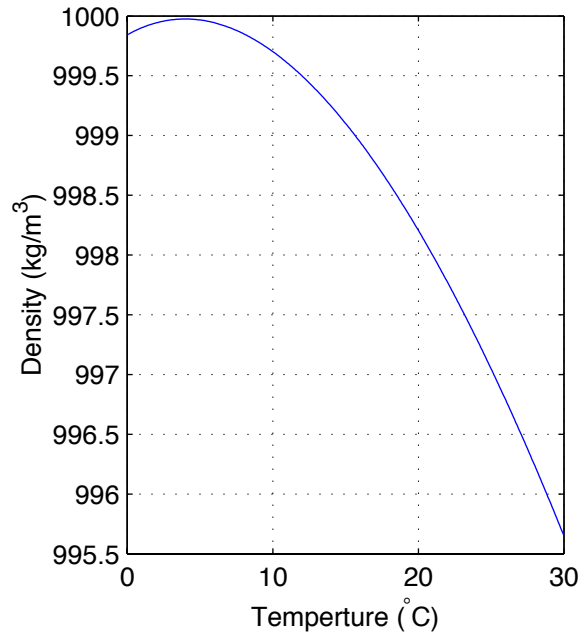
- **Density** $\rho = \frac{\text{Mass}}{\text{Volume}}$

There is less than a 1% change in the density of water over the standard range of temperatures seen in the environment yet this difference can be very important!

1.5.1 Specific Weight

$$\gamma = \rho g = \frac{\text{weight}}{\text{volume}}$$

$$\gamma_{\text{water}} = 62.4 \text{ lbs/ft}^3$$



1.5.2 Specific Gravity

The specific gravity is the density of a substance normalized by the density of water at a certain temperature, often 4°C, the temperature of maximum density at normal pressures. Hence we write

$$S.G. = \frac{\rho}{\rho_{\text{water @ } 4^{\circ}\text{C}}} = \frac{\rho}{1000 \text{ kg/m}^3} \text{ in S.I. units}$$

S.G. of sands and gravels is about 2.6 - 2.7.

1.6 Perfect Gas Law

$$P = \rho R \Theta$$

where R is the specific gas constant which can be expressed as $R = C_P - C_V$ where C_P is the specific heat at constant pressure and C_V is the specific heat at constant volume.

We also can write

$$R = \frac{\Lambda}{MW_{\text{gas}}}$$

where Λ is the universal gas constant ($8314 \text{ m}^2\text{s}^{-2}\text{K}^{-1}$) and MW_{gas} is the molecular weight of the gas.

Let's check the units

$$P = \frac{[\text{M}]}{[\text{L}^3]} \cdot \frac{[\text{L}^2]}{[\text{T}^2\Theta]} \cdot \Theta = \frac{[\text{M}]}{[\text{LT}^2]} = \frac{[\text{ML}]}{[\text{L}^2\text{T}^2]} = \frac{\text{Force}}{\text{Area}}$$

Example

Find ρ and γ for CO_2 at 100°C .

$$\rho = 1.44 \text{ kg/m}^3; \text{ S.G.} = 14.1 \text{ N/m}^3$$

1.7 Viscosity

$$d_1 = u_1 \cdot dt$$

$$d_2 = u_2 \cdot dt = (u_1 + du)dt$$

$$\text{Strain} = \frac{d_2 - d_1}{dz} = \frac{(u_1 + du - u_1)dt}{dz} = \frac{du}{dz}dt$$

For solids we know that stress is proportional to strain. In fluids we find that *stress is proportional to strain rate*.

$$\text{Strain rate} = \frac{\frac{du}{dz}dt}{dt} = \frac{du}{dz} \Rightarrow \text{the velocity gradient is the strain rate!}$$

Therefore since

$$\text{stress} \propto \text{strain rate} \quad (1.1)$$

$$\tau \propto \frac{du}{dz} \quad (1.2)$$

$$\tau = \mu \frac{du}{dz} \quad (1.3)$$

Therefore

$$\mu = \frac{\tau}{\frac{du}{dz}} = \frac{[\text{MLT}^{-2}\text{L}^{-2}]}{\frac{[\text{LT}^{-1}]}{[\text{L}]}} = \frac{[\text{M}]}{[\text{LT}]}$$

what is this? Momentum has the units of mass times velocity hence we can interpret μ as having the dimensions of momentum per area. Thus we can think of μ , known as the *viscosity*, as the amount of momentum transported by molecular activity across a given area. Thus highly viscous fluids (honey) transport lots of momentum and tend to be harder to move (be more sticky) as you have to move the whole fluid while low viscosity fluids (water) tend to be easier to move as only a small parcel of fluid is affected by trying to move a thin slab of fluid.

1.7.1 Kinematic Viscosity

If we normalize the viscosity by the density we have the *kinematic viscosity*.

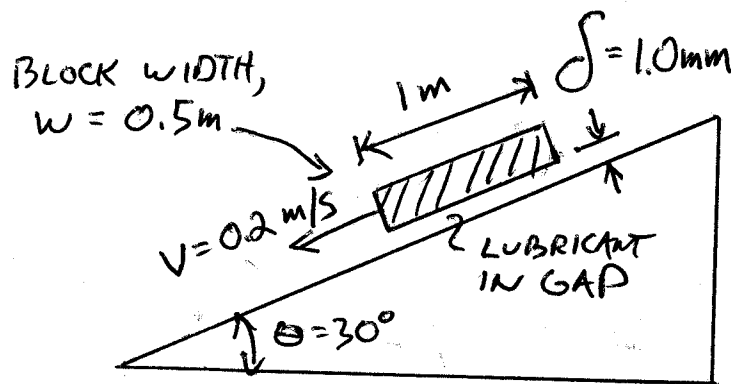
$$\nu = \frac{\mu}{\rho} = \frac{[\text{L}^2]}{[\text{T}]}$$

At 20°C water has an absolute or dynamic viscosity of $1.0 \times 10^{-3} \text{ N s m}^{-2}$ (or Pa s) and a kinematic viscosity of $1.0 \times 10^{-6} \text{ m}^2\text{s}^{-1}$.

Now, if we have a thin gap filled with a fluid but the solid surfaces on either side of the gap have some relative velocity (e.g., one surface is fixed but the other is moving) then there will be stress on either solid surface transmitted by the fluid. The molecules on either solid boundary must be moving at the speed of the boundary, this is known

as the *no-slip boundary condition*. If the fluid filled gap is long compared to its width then we can ignore what happens at the end of the gap and, if the system is at steady state (meaning the velocity profile of the fluid in the gap is no longer changing in time) then we would find that the velocity profile just varies linearly, going from the velocity of the one boundary to the velocity of the other boundary. We will actually solve for the exact solution from the equations of motion later in the semester! A linear velocity variation is a constant velocity gradient hence the fluid stress is constant, just equal to the fluid viscosity times the constant velocity gradient. This is perhaps best illustrated by an example.

1.7.2 Example - A block sliding down an inclined plane



If the block has mass 1 kg:

1. Determine the viscosity, μ , of the lubricant fluid in the gap.
2. What speed will the block travel if the angle, θ , is adjusted to 10° and the gap, δ , is decreased to 0.5 mm

$$1) \mu = 0.049 \frac{\text{kg}}{\text{m} \cdot \text{s}} \left(= \frac{\text{N} \cdot \text{s}}{\text{m}^2} = \text{Pa} \cdot \text{s} \right)$$

$$2) V = 0.070 \frac{\text{m}}{\text{s}}$$