1.11 Review

- $\tau = \mu \dfrac{du}{dz}$ where in a small gap we assume $\dfrac{du}{dz}$ is a constant (e.g., the velocity profile is linear) $\Rightarrow$ therefore $\dfrac{du}{dz} = \dfrac{\Delta u}{\Delta z}$

- Vapor pressure: High velocity $\Rightarrow$ low pressure $\Rightarrow$ cavitation.

- Surface tension: ΔP is inversely related to R - the surface curvature (e.g., for bubble we found $\Delta P = 2\pi/R$).

1.12 The Velocity Field - From Two Perspectives

The *Eulerian* velocity is the velocity with respect to a fixed position. E.g., you stand on a hill with a velocity sensor and record the three components of velocity as a function of time. This is the Eulerian velocity vector at that physical point. We write the Eulerian velocity as

$$\vec{v} = \vec{v}(x, y, z, t) = u(x, y, z, t)\vec{i} + v(x, y, z, t)\vec{j} + w(x, y, z, t)\vec{k}$$

where $\vec{i}$, $\vec{j}$, and $\vec{k}$ are the unit normal vectors in the Cartesian x, y, and z directions, respectively.

The *Lagrangian* velocity is the velocity with respect to a point following the flow. E.g., you jump on a parcel of fluid at the hill (say your starting point is (x_0, y_0, z_0)) and record your velocity with respect to a reference frame (in this case a point on the top of the hill, perhaps). We write the Lagrangian velocity as:

$$\vec{v} = \vec{v}(x_0, y_0, z_0, t) = u(x_0, y_0, z_0, t)\vec{i} + v(x_0, y_0, z_0, t)\vec{j} + w(x_0, y_0, z_0, t)\vec{k}$$

1.13 This Line and That Line: The Pathline, Streamline, and Streakline

Now that we have described the velocity field we can think of various ways to define lines through the velocity field, each with its own significance.

- *Pathline* – the path traced out by a fluid parcel. E.g., the Lagrangian track of parcels of fluid.
- *Streamline* – A line instantaneously tangent to the velocity field. E.g., if you could freeze the velocity field at an instant in time this is the path a vehicle following the velocity vectors would trace out.
- *Streakline* – the locus of parcels that have previously passed through a given point. E.g., if you rotate a hose from left to right (around a fixed point) the jet of water as it exists in the air is a streakline.

How do we analytically calculate these various lines?

Pathline

$$
\begin{aligned}
x &= \int u(x_0, y_0, z_0, t)\, dt \\
y &= \int v(x_0, y_0, z_0, t)\, dt \\
z &= \int w(x_0, y_0, z_0, t)\, dt
\end{aligned}
$$

Streamline

$$\text{Tangent}=\frac{d\vec{r}}{\vec{v}} = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

Streaklines

A mathematical challenge!

1.13.1 Example 1.12 from the textbook

1.13.2 A Streakline Example