2.9 Review

- Pressure measurement – *absolute*, relative to a vacuum; *gage*, relative to local atmosphere; *vacuum/suction*, the negative of gage.
- $P_{\text{down}} = P_{\text{up}} + \gamma|\Delta z|$
- If the surface $\mathcal{S}$ is horizontal, $P = a\ constant \quad \Rightarrow \quad F_R = PA$
- If the surface is not horizontal $\quad \Rightarrow \quad F_R = P_c A$ where P_c is the pressure at the centroid.
- The location of F_R is **below** the centroid, always!

2.10 Hydrostatic Force on a Curved Surface

There are two approaches:

1. $dF_R = P\,dA \quad \Rightarrow \quad$ non-planar hence the integration is hard!
2. Apply a static control volume

$$\sum F_x = 0 = F_{\text{AC}} - F_{\text{H}} \quad \Rightarrow \quad F_{\text{AC}} = F_{\text{H}}$$

$$\sum F_z = 0 = -F_{\text{AB}} - W + F_{\text{V}} \quad \Rightarrow \quad F_{\text{V}} = F_{\text{AB}} + Q$$

Where is the center of pressure?

F_{H} <u>must</u> be collinear (no shear) with F_{AC}

F_{V} <u>must</u> be collinear with the resultant of $F_{\text{AB}} + W$

Example – Oil Tanker

$$F_{\mathrm{H}} = F_{\mathrm{AC}} = P_C A = \gamma h_C A = \gamma(d - \frac{r}{2})rb$$

Now assume we are interested in solving for a unit width (i.e., $b = 1$ m). Therefore

$$F_{\mathrm{H}} = 10\frac{\mathrm{kN}}{\mathrm{m}^3}(24\mathrm{m} - \frac{1.5\mathrm{m}}{2})(1.5\mathrm{m})(1\mathrm{m}) = 349\mathrm{kN}$$

2.10.1 Line of Action

How do we determine the line of action for the resultant force on a curved surface? We could use our moment of inertia based method reviewed above but let's use pressure prisms here.

$$F_1 = \gamma(d - r)rb = 10 \cdot 22.5 \cdot 1.5 \cdot 1 = 337.5\mathrm{kN}$$
$$y_1 = d - \frac{r}{2} = 23.25\mathrm{m}$$
$$F_2 = \gamma\frac{r}{2}rb = 10 \cdot \frac{1.5}{2} \cdot 1.5 \cdot 1 = 11.25\mathrm{kN}$$
$$y_2 = d - \frac{r}{3} = 23.5\mathrm{m}$$

Therefore

$$y_{\mathrm{AC}}F_{\mathrm{AC}} = F_1 y_1 + F_2 y_2 \quad \Rightarrow \quad y_{\mathrm{AC}} = \frac{337.5\mathrm{kN} \cdot 23.25\mathrm{m} + 11.25\mathrm{kN} \cdot 23.5\mathrm{m}}{348.75\mathrm{kN}} = 23.26\mathrm{m}$$

Similarly

$$F_V = 355.3\mathrm{kN} \quad \text{and} \quad x_{\mathrm{BC}} = -0.756\mathrm{m} \quad (x = 0 \text{ at the tanker's vertical surface})$$

2.11 Pressure Prism

Consider the example above:

We can solve this directly as just shown, however, for many situations (or just for many people who prefer to think in a different manner!) a decomposition of the pressures into a series of *pressure prisms* is often easier. Consider the decomposition such that

$$\begin{aligned} F_{y_R} &= F_1 y_1 + F_2 y_2 \\ &= (\gamma b h_2 h_1)\left(h_1 + \frac{h_2}{2}\right) + \left(\gamma b \frac{h_2^2}{2}\right)\left(h_1 \frac{2h_2}{3}\right) \\ \text{Therefore } y_R &= \frac{h_1\left(h_1 + \frac{h_2}{2}\right) + \frac{h_2}{2}\left(h_1 + \frac{2h_2}{3}\right)}{h_1 + \frac{h_2}{2}} \\ &= h_1 + \frac{h_2}{2}\left(\frac{h_1 + \frac{2h_2}{3}}{h_1 + \frac{h_2}{2}}\right) \\ &= 4 + 3\left(\frac{4+4}{4+3}\right) = 7.43' \end{aligned}$$