A bonus example BOD problem:
In a lab measurement of BOD, 10 mL of a wastewater is diluted to 300 mL. The initial D.O. level (after dilution) is 10 mg/L. The test bottle is incubated at 20 °C, but after 3 days the incubator malfunctions, and the temperature changes to 25 °C. After 5 days, a D.O. level of 4 mg/L is measured. If the 20 °C rate constant \( k \) is 0.2/day and \( \theta = 1.05 \), find the correct BOD\(_5^{20^\circ C}\) of the wastewater.

Solution:
1) Formulate a conceptual picture. What happened to D.O. in the bottle vs. time?

If we knew \( L_0 \), we could solve the problem, since:

\[
\text{BOD}_5 = L_0 (1 - e^{-k \cdot 5})
\]
For the first 3 days, $k = 0.2$/day (at $20 \, ^\circ C$), but for the next two days, $k$ will be different because $T = 25 \, ^\circ C$.

\[
k_{T=25C} = k_{T=20C} \theta^{(25-20)} = 0.2 \times (1.05)^5 = 0.255/\text{day}
\]

After 3 days, the BOD exerted is: $y_3 = L_0 (1-e^{-kt}) = L_0 (1-e^{-2 \times 3}) = 0.45L_0$

After 3 days, the BOD concentration remaining is $L_3 = L_0 e^{-2 \times 3} = 0.55 \, L_0$
This is the starting BOD$_L$ for the next two days ($L_3$).

After the next 2 days, the BOD exerted is: $y_2 = L_3 (1-e^{-255*2}) = 0.4L_3$

And, since $L_3 = 0.55 \, L_0$, $y_2 = 0.4 \times (0.55L_0) = 0.22 \, L_0$

Over the entire 5 days, the BOD exerted = \Delta \text{D.O.} = 10 - 4 = 6 \, \text{mg/L},
and the BOD exerted = $y_3 + y_2 = 0.45L_0 + 0.22L_0 = 0.67L_0$

So, $0.67 \, L_0 = 6 \, \text{mg/L}$, and $L_0 = 6 / 0.67 = 9.0 \, \text{mg/L}$ (for the diluted waste)
$L_0 = 9 \times (300/10) = 270 \, \text{mg/L}$ (for the full-strength waste)
Finally, we want to know the correct BOD$_5$ at 20°C

\[ y_5 = L_0(1 - e^{-k*5}) \]
\[ = 270(1 - e^{-0.2*5}) \]
\[ = 170 \text{ mg/L} \]