A comparison of PFR and CSTR reactor performance

Based on an old exam problem:
(a) Calculate the fraction of contaminant remaining \( \left( \frac{C}{C_o} \right) \) in the effluent of a PFR with a hydraulic retention time of 10 days. Assume the contaminant obeys 1st order kinetics with a rate constant of 0.2/day. Assume a continuous input.

For a PFR:
\[
\frac{C}{C_o} = e^{-kt} = e^{-0.2/10\text{d}} = 0.135
\]
b) Calculate the fraction of contaminant remaining \((C/C_o)\) in the effluent of a CSTR with a hydraulic retention time of 10 days. Assume the contaminant obeys 1\textsuperscript{st} order kinetics with a rate constant of 0.2/day. Assume a continuous input.

For a CSTR:

\[
\frac{C}{C_o} = \frac{1}{1 + k(V/Q)} = \frac{1}{1 + \frac{0.2}{10 \text{d}}} = 0.33
\]
c) What is the least number of CSTR’s in series that it would take to achieve a fractional contaminant removal within 10% of part (a)? Assume the CSTRs are of equal size and that their total volume is the same as the PFR. Assume the contaminant obeys 1st order kinetics with a rate constant of 0.2/day. Assume a continuous input.

First consider 2 CSTRs in series:

\[
\frac{C_1}{C_o} = \frac{1}{1 + k(V/Q)}, \quad \text{and} \quad \frac{C_2}{C_1} = \frac{1}{1 + k(V/Q)}
\]

so:

\[
\frac{C_2}{C_o} = \frac{C_1}{C_o} \times \frac{C_2}{C_1} = \left(\frac{1}{1 + k(V/Q)}\right)^2
\]

for n CSTR's in series:

\[
\frac{C_n}{C_o} = \left(\frac{1}{1 + k(V/Q)}\right)^n
\]

for n CSTR's in series with a constant total volume:

\[
\frac{C_n}{C_o} = \left(1 + k\frac{(V/Q)}{n}\right)^n
\]
We want \( \frac{C_n}{C_o} \leq 1.1(0.135) = 0.149 \)

\[
0.149 = \left( \frac{1}{1 + 0.2 \left( \frac{10}{n} \right)} \right)^n
\]

Solve by trial:

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \frac{C_n}{C_o} )</th>
</tr>
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<tr>
<td>3</td>
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<tr>
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<td>20</td>
<td>0.149</td>
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