Problem Set No. 7

Problem 1.
Assume that the BOD removal rate can be described by the following equation that is first order with respect to both cell concentration (X) and BOD concentration (S): \[-\frac{dS}{dt} = kXS\], where: \( k = \) constant (1/day). Also, assume that cell growth rate is directly proportional to the rate of BOD removal: \[-\frac{dX}{dt} = -Y \frac{dS}{dt}\], where: \( Y = \) yield coefficient (mg cells/mg BOD consumed). And assume that cell death is first order with respect to cell concentration: \[-\frac{dX}{dt} = -bX\], where: \( b = \) a constant (1/day).

Derive an equation to predict the steady state effluent BOD concentration (S) from a CSTR if: \( S_o = \) influent BOD concentration (mg/L); \( Q = \) influent flow rate (L/day); \( V = \) CSTR reactor volume (L); \( X_o = \) influent cell concentration = zero; and the CSTR has no recycle flow.

Problem 2.
Assume: A batch reactor (Q = 0)
\[ k = 10 \text{mg BOD}_L/\text{mg X/day} \]
\[ K_s = 35 \text{mg BOD}_L/L \]
\[ Y = 0.65 \text{mg X/mg BOD}_L \]
\[ b = 0.10 \text{day}^{-1} \]

a) If the food supply is unlimited and large (promoting exponential growth), and the initial concentration of bacteria is 0.25 mg X/L, then what will be the biomass concentration (in mg cells/L) at the end of three days? \textbf{Ans.:} 5.45 \times 10^7 \text{mg cells/L}

b) Bacteria multiply by binary fission, doubling their number with each new generation. Calculate the "generation time" (in days) for this system (i.e., the length of time it takes for the population to double). \textbf{Ans.:} 2.59 \text{ hours}

Problem 3.
Adapted from Environmental Science and Engineering by Henry & Heinke.

A 1000-m$^3$/day activated sludge plant was designed for food to microorganism ratio (F/M) of 0.3 day$^{-1}$ and BOD$_5$ reduction in the aeration system from 150 mg/L to 15 mg/L. The concentration of MLVSS of 2000 mg/L in the aeration tank can be increased to .95% in the secondary settling tank and the settled biomass is recycled to the aeration tank at an average rate 258 m$^3$/day. See schematic on the following page.
a. What is the aeration tank volume V and hydraulic detention time (θ)?

b. What is the total mass of activated sludge removed from the system each day (in the effluent, plus the waste sludge)? [Hint: consider a mass balance on cells in the aeration tank. At steady state in the aeration tank $V \frac{dX}{dt} = 0$ and the cell production rate (mass/day) must be equal to the mass of cells removed per day.] Ans.: 65 kg/day

c. What is the specific utilization rate $\mu = \frac{dX}{dt} X$? [Note, $\mu$ is also the mass of volatile solids produced each day, divided by the mass of volatile solids in the system.] Ans.: .13/day

d. What is the specific substrate removal rate $-\frac{dS}{dt} X$? Ans.: .27/day

e. What is the cell yield (Y)?

f) What is the cell retention time (θc)? Ans.: 7.7 day

Problem 4.

A soluble industrial waste is to be aerobically treated:

- $Q = 10^7$ L/day
- $S_0 = 200$ mg BOD/L
- $Y = 0.6$ g VSS/g BOD
- $k = 6$ g BOD/g VSS/day
- $K_s = 75$ mg BOD/L
- $b = 0.1$ day$^{-1}$

Design a CSTR (with recycle) activated sludge system to achieve at least 95% removal of influent BOD with an adequate safety factor. Use an MLVSS concentration of 3000 mg/l in your design and assume a reasonable value of $X_r$. Specify solids retention time, hydraulic retention time, recycle ratio, and reactor volume. Also specify sludge wasting rate (kg/day of VSS requiring disposal).

Problem 5. (from the 1994 Final exam)

Consider a conventional activated sludge reactor system. In this problem you are asked to evaluate the extent of nitrogen removal.
Given a) Influent characteristics: $BOD_L = 150 \text{ mg/L}$; Dissolved Nitrogen $= 30 \text{ mg/L}$
   b) Solids retention time $= 5 \text{ days}$
   c) Maximum specific substrate utilization rate $= 10 \text{ g } BOD_L / \text{g VSS} \cdot \text{day}$
   d) Specific decay rate for VSS $= 0.1 / \text{day}$
   e) Half-velocity coefficient $= 50 \text{ mg } BOD_L / \text{L}$
   f) Yield coefficient $= 0.55 \text{ g VSS/g } BOD_L$
   g) Nitrogen content of biomass $= 0.124 \text{ g N/g VSS}$ (i.e. $C_5H_7O_2N$)

Find: the dissolved nitrogen concentration in the effluent from the secondary clarifier.
A schematic of the reactor is shown below.

You may assume:
   a) The $BOD_L$ concentration in the plant effluent is very small,
   b) steady state conditions,
   c) the cell concentration in the secondary effluent is negligible.

Ans.: $N = 23.2 \text{ mg/L}$

Problems 6.
[From the 1987 Final Exam] One of the modifications of the conventional activated sludge process is “Step Aeration”; a schematic of this process is shown below:

We haven’t developed equations for AFR activated sludge systems, however, as a first approximation we can model this system as a series of two CSTRs as shown below.
a) Write a governing equation for the cell retention time ($\theta_c$) for the system shown above.

b) Given the following physical and biological parameters relevant to the system, find the steady-state effluent $S$ (mg BOD$_L$/L).

\[
\begin{align*}
Q &= 1 \text{ MGD} \ (3.79 \times 10^6 \text{ L/day}) \\
V_1 &= V_2 = 1.6 \times 10^5 \text{ L} \\
Q_w &= 7 \times 10^4 \text{ L/day} \\
r &= 0.3 \\
\text{Influent BOD}_L &= 300 \text{ mg/L} \\
Y &= 0.5 \text{ g VSS/g BOD}_L \\
b &= 0.1 \text{ day}^{-1} \\
k &= 12 \text{ g BOD}_L/\text{g VSS \cdot day} \\
\text{K}_S &= 40 \text{ mg BOD}_L/\text{L} \\
\text{Influent } X &= 0 \\
X_1 &= 3000 \text{ mg/L} \\
X_2 &= 2000 \text{ mg/L} \\
X_r &= 8000 \text{ mg/L}
\end{align*}
\]

Ans. $S = 27.6 \text{ mg BOD}_L/\text{L}$

Problem 7.
Do STELLA Exercise #6. Don’t forget to turn your STELLA exercise in separately.