STELLA Assignment #2 - BOD

1) In this problem, you will be tracking the biochemical oxygen demand (BOD) impact of a waste discharged from the Watapiti waste facility. The plant discharges 7.5 x 10^5 liters/day of waste (Q_p). The raw waste coming into the facility has an ultimate BOD (BOD_L) concentration of 250 mg/L. The concentration leaving the plant in the effluent (BOD_P) depends on the efficiency of the plant (E):

\[
BOD_p = 250 \times (1 - E) \text{ mg/L}
\]  
(1)

E is a fraction between 0 and 1 (change the percentage efficiency to a decimal). Sadly, the facility works at a meager efficiency of 45% (E = 0.45).

Once discharged to the stream, the treated wastewater mixes with the stream, which has a flow of Q_S = 7.5 x 10^6 liters/day. Downstream from Watapiti, the total river flow will be Q_S plus the effluent from the treatment plant (Q_P). For the time being, we will assume the stream has zero BOD before the wastewater is added.

As discussed in class, the following equation may be used to solve for BOD downstream:

\[
BOD_t = BOD_o \times e^{-kt}
\]  
(2)

where: BOD_t is the BOD concentration (in mg/L) at t time units away from the source
- BOD_o is the initial BOD in the river at the point of discharge after mixing (mg/L)
- t is the time (days)
- k (or k_d) is the degradation constant (day^{-1})

The rate constant for BOD loss can be expressed as the sum of different types of BOD loss. For our model, we will consider decay due to biological removal (k_1) and loss by sedimentation (k_3). For now, we will consider k_1 = 0.24/day and k_3 = 0.03/day. Assume the velocity (U) of the river above and below Watapiti is 5 km/day.

Calculate (using equation 2 above, not STELLA) the BOD concentration at 0, 10, and 20 km downstream from the Watapiti treatment plant. The distance downstream (X) is the velocity (U) multiplied by the time (t) traveled. Remember at distance 0 km to calculate the initial concentration by mixing the effluent with the stream water.

**Partial answer:** BOD_{X=20 \text{ km}} = 4.24 \text{ mg/L}

2) Now you will use STELLA to answer the same BOD problem. You will want to start with a copy of the STELLA model you created for decay of a radioactive material. The introduction to STELLA and the first STELLA assignment will probably be useful.
Remember the radioactive source? The amount of the source was undergoing an exponential decline, just as the BOD concentration declines exponentially. In fact, a STELLA model for BOD would be very similar to the model that you developed for the radioactive source. Using STELLA, develop a BOD model. You will not use the exponential formula for decay of BOD concentration that you used to answer the preceding question. STELLA performs a numerical integration of the BOD rate equation, \[ \frac{dBOD}{dt} = -(k_1 + k_3)BOD, \] as follows:

\[
BOD(t) = BOD(t_{-dt}) + (-\text{Decay\_Rate}) \times dt \tag{3}
\]

where:

\[
\text{Decay\_Rate} = \text{Fraction\_Decaying} \times BOD
\]

and

\[
\text{Fraction\_Decaying} = k_1 + k_3
\]

Note that STELLA automatically assigns a negative sign to a flow out of a stock. Thus, Decay\_Rate is set to be negative by STELLA since it is a flow out of BOD.

Thus the change in the BOD concentration with respect to time is found by multiplying the degradation rate constant times the current BOD concentration. This is exactly like the SOURCE amount that was changing at a rate equal to a constant fraction times the current amount. To develop the BOD model for the Watapiti plant in Part 1, simply follow the same procedure as for the model for radioactive decay, changing the names and values as necessary. In fact you may want to open up your first STELLA homework and use the Save As... command under the File menu to create a copy with a new name (BOD ??) that you can then modify for doing this assignment.

What was formerly the SOURCE stock can now be the stock for BOD. If you are using the former RAD.SOURCE model, please rename the stock to avoid confusion. Make the initial value of BOD equal to the BOD\_o value you calculated earlier in Part 1. Since both k_1 and k_3 can vary, it is desirable to keep them as separate converters. This can be done by adding two converters to the model and connecting them to the previous converter FRACTION DECAYING. FRACTION DECAYING may then be defined as equal to: \( k_1 + k_3 \) \( \text{day}^{-1} \).

Once you have a model diagram, set up a graph of BOD. Click on the GRAPH PAD icon and deposit it on your model diagram. Double click on the untitled graph that appears, and select BOD from the list of Allowable items. Name the graph (BOD?). Set the Y-axis scale for BOD to have a minimum of 0 and a maximum of 14 mg/L. Click on OK. Use Run Specs in the RUN menu to set the simulation time to 10 days. Also adjust the time increment (DT) that the model uses to 0.01 days to help us check your answer. Remember that the most accurate answer would be one derived from an infinite number of time steps.

Using the Table icon, create a table. Double click on the untitled table that appears, and select BOD from the list of Allowable items. When you are defining the table, change the report interval from every DT to 1.0 (i.e., once per day). You do not need to see the BOD every time
increment. Name the table (BOD Table??).

Return to the model window and switch to the equation view. Be sure that you have defined all the variables so that no "?s" appear (otherwise the model will not run). Then, try running your model. Go to the GRAPH PAD. Does BOD exhibit an exponential decay? (If it does not, you did something wrong. Review the radioactive source model again. The model is the same; the names and values have been changed.)

It is a relatively simple matter to add distance downstream to your model. Go to the model window. Use the mouse to click on the converter symbol and deposit it on the screen. Label this converter, VELOCITY. Double-click on the VELOCITY converter and define it with the constant value given in Part 1. Switch back to the Model view and place another converter on your screen near the VELOCITY converter. Label this one DISTANCE. Use the arrow to draw a connection from the VELOCITY to DISTANCE. Note the following:

\[
\text{Distance} = \text{Velocity} \times \text{Time} \quad [\text{km} = (\text{km/day}) \times \text{day}] 
\]  

Double-click on the DISTANCE converter. To define distance, click on VELOCITY from the allowable list. Then click on the multiplication symbol (*), or simply type it in using the keyboard. Then, from the list of BUILT-INS click on TIME (You may have to scroll down the list to find TIME). You should end up with the equation listed above. Click on OK.

Now go to the Define Table dialog. Click-on from the list of Allowables and add it to the table. Make sure your table has a vertical orientation. Click on OK.

Run your model again. Get a printout of the Table. You should now be able to use the STELLA model to answer the questions in Part 1. Does your model calculate nearly the same values as you did by hand in Part 1? (Greater variances may mean a problem with either your hand calculations or the STELLA model you developed.) STELLA is a numerical simulation, and is getting close to the exact values you calculated. STELLA can use other simulation techniques that may come closer to the exact numbers (see Integration Method using Time Specs (Run Specs in Windows)... under the RUN menu), but they take longer to run.

It would be nice if STELLA calculated the BOD\(_0\) for you. To do this, you must add a few more converters to your model. By clicking on the converter, depositing it, and naming it appropriately add the following converters to your screen: EFFICIENCY, BOD, and BOD P. (WASTE_BOD stands for the original waste concentration coming into the plant, BOD P is the BOD concentration of the treated plant effluent.) Position the converters near each other. (To move a variable on the screen that was already deposited on the screen, a single click-and-hold
on the variable will allow you to move it.) Double-click on the EFFICIENCY and WASTE_BOD converters. Enter the constant values given in Part 1. (Remember to enter EFFICIENCY as a decimal.) Then draw connections from EFFICIENCY and WASTE_BOD to P. Then double-click on BOD_P. Define it as follows:

\[ \text{BOD}_P = \text{WASTE}_BOD \times (1 - \text{EFFICIENCY}) \text{ (mg/L)} \]  

(5)

Click on the ▶ symbol on the left of the dialog box to close it.

Add the following converters anywhere on your screen: \( Q_P \) and \( Q_S \) (the plant flow and the upstream flow in L/day, respectively). Double-click on each and define them as in Part 1. In STELLA, you can enter values in scientific notation by using the letter ‘E’ (e.g., 7.5E5).

Now you have enough converters to make the INITIAL value of BOD equal to the algebraic equation for mixing with the stream water. To do so, double-click on the BOD stock. You are going to create the mixing relationship in the equation box. Click on the model menu (not the model view tab) and click on find. You will see a list of the converters you have created including \( Q_S \), \( Q_P \), and \( \text{BOD}_P \). You may drag the icon for any needed converter into the equation box. Drag \( \text{BOD}_P \) from the Find list, select * from the mathematical operators, then drag the icon for \( Q_P \). Next select / for division from the mathematical operators and a left hand parenthesis. Drag \( Q_P \) from the Find list, select + from the mathematical operators, drag \( Q_S \) from the find list, and then select a right hand parenthesis. Now look at the formula you have created. This should look like the mixing formula you used to calculate \( \text{BOD}_0 \) by hand. Close the definition box by clicking on the arrow at the middle of its left hand side.

Look over your equation view. Any ?’s left? Make sure every unit is defined, either by a constant value or an algebraic formula. Now run your model again. Go to the table window; your values should not have changed. Make a hard copy of your diagram, equations, Run Specs dialog, the results shown in your table and print out your graph (remember that the number of pages you print can be reduced by pasting your model diagram, equations, graph and a copy of the values in your table into a document file of a word processing program (such as MS Word), where some of the objects can be resized). Compare your results to those determined in part 1. Be sure to clearly indicate your solution to the questions (i.e., don’t just hand in a print out of a table and expect the grader to read your mind to figure out what number(s) you think are important). Remember to save your BOD model.

3) Using your STELLA model, change the plant efficiency to 80%. What will the BOD concentration be at 0, 10, and 20 km downstream from Watapiti? Change the maximum of your BOD graph to 6 mg/L. Make a hard copy of the Table and Graph Pad for these results. Once again please be sure to clearly indicate your solutions to the questions (i.e., please don’t hand in a print out of an unmarked table and expect the grader to read your mind to figure out what number(s) you think are important). Partial answer: \( \text{BOD}_{@X=10\ km} = 2.65 \text{ mg/L} \)
Approximately how low can the plant efficiency be (to the nearest 1%) while keeping the river concentration of BOD at 10 km below 5 mg/L? Include a print out of data from STELLA to defend your answer.

If there is a drought in Watapiti, and the upstream flow ($Q_s$) drops to $5 \times 10^6$ liters/day, approximately how efficient (to the nearest 1%) must the treatment be to maintain this same standard? Include a STELLA printout to defend your answer.

4) There are many alternative kinetic models for bacterial assimilation of organic matter (you will see others as we make progress in this course). The Monod model approximates bacterial kinetics in a manner analogous to enzyme kinetics. In this model, the rate of degradation is not always first order. The rate equation for Monod kinetics has the following form:

$$\frac{-d[BOD]}{dt} = \frac{kX[BOD]}{K_s + [BOD]}$$

(Equation 6)

where:
- $[BOD]$ is the concentration of organic waste (mg/L)
- $X$ is the concentration of bacteria
- $k$ is the maximum utilization rate ($\text{time}^{-1}$)
- $K_s$ is referred to as the half velocity constant (mg/L).

Note that, when $[BOD]$ is much smaller than $K_s$, and the cell concentration, $X$, stays constant, then equation 6 above becomes first order;

i.e.: $$\frac{-d[BOD]}{dt} = \frac{kX[BOD]}{K_s + [BOD]} \approx k_d[BOD]$$

(Equation 7)

Let’s construct a STELLA model that uses Monod kinetics and compare the results to the first order degradation kinetics that you have just finished modeling. Your new model will need converters for $K_s$ and $kX$ (for now, we will continue to assume the cell concentration stays constant). In the above simulation, you used a $k_d$ value of 0.27/day (i.e., $k_1 + k_3$). Assume $K_s$ is equal to 35 mg/L, and calculate a $kX$ value so that $k_d$ stays equal to 0.27 in equation 7. Run the revised model for the case where plant efficiency was 45% (and $Q_s=7.5 \times 10^6$) and compare it to your prior results. Hint: just make a new stock called BOD_MONOD, and hook up the required flow and converters. (Be sure to use equation 6, not equation 7, to define the fraction decaying for the Monod model. Also remember that a flow out of a stock causes it to decrease and accounts for the negative sign in the equation.) Then modify your graph so that it plots both BOD and BOD_MONOD. Make sure that Y-axis scale for both variables has the same maximum (so that the entirety of both curves is shown) and minimum (zero). Extend the length of the simulation to 20 days. The new plot will directly let you compare results. Add BOD_MONOD to your table.
As noted above, the rate of bacterial assimilation of BOD can also be dependent on the concentration of microorganisms. Equation 7 above assumes that the bacterial population is not changing significantly. Later in this course, we will develop models where the concentration of microorganisms is treated as a variable.

Turn in a printout of your diagram, equations, Run Specs dialog, a graph of the simulation results and a table for the comparison. A summary of everything you need to turn in is given below.

<table>
<thead>
<tr>
<th>Summary of STELLA output and information you should turn in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASE</td>
</tr>
<tr>
<td>Calculations using the BOD equation</td>
</tr>
<tr>
<td>45% Efficiency</td>
</tr>
<tr>
<td>80% Efficiency</td>
</tr>
<tr>
<td>? efficiency for a BOD = 5 mg/L @ x=10 km</td>
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<tr>
<td>? efficiency for a BOD = 5 mg/L @ x=10 km @ low Q_s</td>
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<tr>
<td>(45% Efficiency) Compare BOD vs. time for models w/1st order kinetics &amp; Monod kinetics</td>
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</tbody>
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