STELLA Assignment #3 - Dissolved Oxygen and BOD

Now that you have a good grasp of the STELLA basics, let's begin to expand the BOD model developed in the past assignment. One concern of an environmental engineer is in assessing the impact on the concentration of dissolved oxygen in a stream that is receiving a waste load.

**Part 1** Dissolved oxygen (D.O.), like BOD may be depicted as a stock in the STELLA model. D.O. may then be depleted and regenerated by use of the flow icons.

The rate of change of dissolved oxygen (D.O.) with time is given by:

\[
\frac{d}{dt} \text{D.O.} = k_2 (C^* - \text{D.O.}) - k_1 \text{BOD}(t)
\]  

where:
- \( k_2 \) is the rate constant for reaeration (a mass transfer coefficient).
- \( C^* \) is the saturation (equilibrium) concentration of BOD at the given water temperature.
- \( \text{D.O.} \) is the instantaneous concentration of dissolved oxygen.
- \( k_1 \) is the rate of bacterial consumption of BOD.
- \( \text{BOD}(t) \) is the instantaneous concentration of BOD (recall, \( \text{BOD}(t) = BOD_0 e^{-(k_1 + k_2)t} \)) as modeled in the previous assignment.

and \( t \) is the time in days.

If you wished to obtain an analytical solution to equation (1) it would be convenient to employ the following equation for dissolved oxygen deficit (D):

\[
D = (C^* - \text{D.O.})
\]  

Then, noting that the oxygen deficit (D) changes at a rate opposite to the rate of change of D.O. allows us to write equation (1) in terms of the deficit:

\[
-\frac{d}{dt} D = k_2 D - k_1 \text{BOD}(t) = k_2 D - k_1 BOD_0 e^{-(k_1 + k_2)t}
\]  

As discussed in class, an exact solution to equation (3) can be found and used to calculate O\(_2\) deficit as a function of time [or its distance equivalent \( X = Ut \), where \( U \) is the stream velocity].

Now we’ll see how to get a numerical solution to equation (1) using STELLA. Let's first model the production of D.O. You'll first want to call up the BOD assignment and enter the Model window. [Note: you won’t need the BOD Monod part of the prior model for this assignment.] Place a stock in the vicinity of the BOD model and label it DISSOLVED OXYGEN (or D.O., or any other appropriate title). An initial value is required to define DISSOLVED OXYGEN (this initial value is the D.O. of the stream after mixing with the effluent). This is calculated by writing a simple mass balance equation. You will need to add a few more converters to the model such as PLANT DO and STREAM DO (the D.O. concentration of the stream before mixing with the effluent). The mass balance that you will employ to calculate the initial D.O. is similar to that used to find the initial BOD.
Assume the effluent from the waste treatment plant contains 1.0 mg/L DO. The amount of dissolved oxygen in a stream depends greatly on what is happening upstream. For your model assume the initial D.O. measured in the stream (before the wastewater input) is 10.0 mg/L.

Now may be a good time to expand your initial BOD equation to allow for the possibility of the stream to contain BOD (again this would depend on the conditions upstream). To do this, you will need to add another converter, STREAM BOD. Then switch to the Model view, double click on the BOD icon and adjust the mass balance equation in the initial value definition box. Let’s assume the STREAM BOD before mixing with the wastewater is 1 mg/L.

**Back to modeling dissolved oxygen.**

The D.O. level in the stream is also dependent on the resupply of oxygen from the atmosphere, or “reaeration”. Reaeration will act to increase the D.O. level. To show a stock being produced at a particular rate, select the flow icon from the diagram view (click once). Position the flow an inch or so to the left of the D.O. stock. Click-and-hold down the button, then drag the icon through to the inside of the D.O. stock and release the click. Label this flow AERATION. Aeration is a function of the saturation D.O. and the reaeration coefficient, $k_2$ in the following form:

$$\text{AERATION} = k_2 \times (C^* - \text{D.O.}) \text{ (mg/L*day)}$$

Where: D.O. is the dissolved oxygen concentration at any time (t); i.e., the contents of your D.O. stock icon. Saturation D.O. (i.e., C*) is a constant in your model (i.e., a converter), but, in reality, it is temperature-dependent. At 10° C, saturation D.O. is 11.28 mg/L (from appendix F in Water Quality, by Tchobanoglous and Schroeder).

The reaeration coefficient is also temperature dependent. In your system, $k_2$ at 20° C is 0.28 day$^{-1}$, and has the following temperature dependency:

$$k_2 \times T = (k_2 \times 20° C) \times [1.025^{(T - 20)}] \times 0.28 \times (1.025^{(T - 20)})$$

Note: use the ^ symbol (shift 6) to raise numbers to a power in STELLA.

You will need to add converters for $k_2$ and C* and connect them to the aeration flow. Also add a converter for Temperature and modify your model to allow the converter, $k_2$, to depend on the Temperature converter. Don’t forget that the reaeration rate depends on the current DO level, so you will need an arrow connecting the DO stock to the aeration flow.

Assume a cool spring day with a water temperature of 10 °C. Dissolved oxygen is depleted by the BOD load on the stream. Add a flow icon to the D.O. stock to account for D.O. depletion. Label it BOD EFFECTS. BOD EFFECTS was represented earlier in the D.O. equation (equation 1, above). As you already know BOD(t) experiences exponential decay and the analytical solution to the BOD rate equation is:
\[
\text{BOD}(t) = \text{BOD}_0 e^{-(k_1+k_3)t}. 
\] (5)

Remember that only bacterial use of BOD effects DO, not removal of BOD by sedimentation (i.e., the $k_1$ part of BOD decay acts to change DO not the $k_3$ part). $k_1$ is already represented in your BOD model, and so is BOD(t). Remember STELLA does not know the equation for exponential decay of BOD (i.e., eq 5). It just knows the expression for the rate of BOD change, i.e., \[ \frac{d(\text{BOD})}{dt} = -(k_1+k_3)(\text{BOD}). \]

BOD(t) is your BOD stock (and STELLA simulates its change over time so that its loss agrees with the governing equation \[ \text{BOD}(t) = \text{BOD}_0 e^{-(k_1+k_3)t} \] ). So to complete your BOD EFFECTS you can simply draw arrows from BOD and $k_1$ to BOD EFFECTS. Be sure to define BOD EFFECTS in the equation view with the proper expression.

For our first simulation let’s assume the treatment plant is operating at 70% efficiency for BOD removal. Before you begin to use the expanded model, let's make sure we all are using the same constants (it is left to the student to check to assure units are in the correct form).

EFFICIENCY = 70%
STREAM FLOW = 5 x 10^6 L/day
PLANT FLOW = 5 x 10^5 L/day
VELOCITY = 5 km/day
WASTE BOD = 250 mg/L
INITIAL STREAM BOD = 1.0 mg/L
INITIAL STREAM DO = 10.0 mg/L
WASTE DO = 1.0 mg/L
SATURATION DO = 11.28 mg/L
TEMPERATURE = 10^\circ C
$k_3$ (rate constant for BOD loss to sediments) = 0.04/day

Previously you used $k_1 = 0.23$ day$^{-1}$ at 20$^\circ$ C. This parameter is also temperature dependent in the following form:

\[
k_1 \text{ at } T = (k_1 \text{ at } 20^\circ C) \times \left[1.047^{(T - 20)}\right] = 0.23 \times (1.047^{(T - 20)}) \] (6)

Make $k_1$ in your model temperature dependent just as you did for $k_2$. Remember you are running the model for a temperature of 10$^\circ$ C. (STELLA hint: if you need to use a converter already placed in the diagram, but it is not at a convenient location, it can be 'duplicated' by using the “ghost” tool.)
Prepare a graph pad showing D.O. and BOD versus TIME. Using an appropriate vertical scale; make the scale the same for D.O. and BOD. Carry out your simulation for 20 days and use a time step (DT) of 0.01 day. Also, expand your table to include DISTANCE, DO and BOD at a 0.2 day report interval. [Review the instructions in prior assignments for the procedures needed to prepare the graph and table pads.]

Run your model for this typical spring day with a stream temperature of 10° C. Look at your graph; do you have what appears to be an oxygen sag curve? Question: Does the dissolved oxygen concentration downstream fall below allowable standards (for example 5 mg/L for trout streams)? Answer this question and make a print of your diagram, graph, equations list and the first page of your table (which should be sufficient to show your value for the minimum D.O.).

Now let's change some of the constants. Consider the situation of a typical late-summer day, where the flow in the stream may be considerably less and the temperature of the stream is 20° C. The stream's flow has dropped to $3 \times 10^6$ L/day. Be sure to change the temperature and the value for D.O. saturation. [See Appendix F of Water Quality by Tchobanoglous and Schroeder for D.O. information. You may assume zero salinity conditions.] Because of the change in D.O. saturation assume that the stream D.O. has dropped from 10 mg/L to 8.5 mg/L upstream of the wastewater plant. We will assume the stream's cross sectional area is such that the flow velocity is unaffected by the change in flow rate. Run the model again; how have these changes affected our stream's quality? Questions: Does the stream still meet minimum standards for D.O. concentration? If not, what wastewater treatment efficiency (to the nearest 0.01) is needed to ensure the D.O. in the stream does not fall below 5 mg/L?

Print out a new graph and table depicting your results for the summer's day. If you wish, change the maximum for D.O. on the Y axis to 10 mg/L (so you can better see where the value of 5 mg/L is located) If adjustments to the treatment plant are required to obtain a minimum DO level of 5.00 ± .02 mg/L, provide a graph and table to prove the problem can be corrected by the proposed new efficiency (and tell us what value you used).

Part 2 The major elements of a stream's oxygen content were incorporated into our STELLA model in the last assignments and part 1 of this assignment, those being BOD and aeration. In this part we are going to add another component hoping to better simulate how a stream assimilates waste.

In part 1 of this assignment, you modeled a BOD input from the waste treatment plant. This is called a point source because the input is limited to a particular location (a point) on the stream. Now suppose there is precipitation occurring within the watershed; it may be of enough
magnitude to cause runoff. This runoff, as it flows along the soil surface, will pick up sediments, nutrients, fertilizers, etc. and carry them to the stream, hence creating another BOD load for the stream to assimilate. This load would be considered a continuous (or nonpoint) source because it would be added “continuously” at the banks along the length of the stream.

Other examples of "continuous" sources could be: in-seepage from groundwater, many small tributaries, or leachate from a nearby waste dump. Keep in mind that the magnitude of these BOD loads at a particular point in space would typically be considerably less than that of a sewage outfall. Let's now add a continuous source of BOD to the STELLA oxygen model. The continuous source is increasing the BOD load onto a stream (i.e., the stock, BOD, is being produced), similar to the way aeration produced DO (with flow icon from 'space' into stock). Label this input CONT SOURCE. You now need to define this new item in your model.

Assume the continuous source load of your stream has a value of 0.5 kg BOD per kilometer*day. For simplicity we will assume this would remain a constant over the length of the stream, but you should note that it likely varies with land applications or use. It also may change with time (a heavy rainfall is likely to produce more of a BOD load than groundwater recharge in the middle of the summer). For simplicity we will also assume that the nonpoint source’s volumetric flow rate has no significant effect on the total flow in the stream ($Q_{TOTAL}$). Your source input is therefore \(5.0 \times 10^5\) mg BOD/(kilometer*day) (provide a converter for this and label appropriately), but this BOD input is into some volume of water. Is it possible to convert this input to units of mg BOD/(liter*day)? [Would we ask you if it wasn't?] Try manipulation of the velocity and flow ($Q_{TOTAL}$) terms. Connect converters for these terms to the continuous source input flow into BOD, and use the equation definition box for the flow to create the needed relationship.

By adding this new item, the equation for BOD has now changed. Previously the equation read:

\[
BOD(t) = BOD(t-dt) + ( - \text{Decay Rate}) \times dt
\]

(STELLA has automatically changed this to accommodate the addition of a continuous source. The equation (which can be seen via the equation view) should now read:

\[
BOD(t) = BOD(t-dt) + (\text{CONT SOURCE} - \text{Decay Rate}) \times dt
\]

You will want to check your method of modeling a continuous source to assure the STELLA model is receiving the proper input. The best way to check is to enter the table window and add CONT SOURCE to your table. When you run the program using the summer’s day flow conditions, the CONT SOURCE column should read a constant 0.71 mg/L*day at all times and/or distances. After your check, remember to delete this from the table; its presence is
uninteresting and really only serves as a check.

In part 1, a graph was developed for the following conditions on a summer’s day:

- STREAM FLOW = 3 x 10^6 L/day
- PLANT FLOW = 5 x 10^5 L/day
- VELOCITY = 5 km/day
- TEMPERATURE = 20 °C (with an appropriate value for saturation DO).

Under these conditions, the waste plant needed a particular efficiency (that you found in part 1) to achieve a dissolved oxygen content of 5.00±.02 mg/L. Run the model with the plant running at that efficiency and with the above input parameters for the nonpoint source. Has the addition of a continuous BOD source caused unacceptable dissolved oxygen levels? If so, what treatment plant efficiency is now required? Show via graphs and tables the conditions that exist before and after any improvements that are required. Be sure to clearly indicate your solutions to the questions (i.e., don’t just hand in a print out of a table and expect the grader to figure out what number(s) you think are important). A table showing what you should hand in is provided below.

How did the addition of the continuous source affect the concentration of dissolved oxygen downstream (at 100 km)? The value at this distance should be close to the steady state (i.e., constant) O_2 level in the stream. Make hard copies of your new diagram and equation windows to turn in with the rest of your assignment.

<table>
<thead>
<tr>
<th>Summary of STELLA output you should turn in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASE</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>10 °C normal flow</td>
</tr>
<tr>
<td>20 °C summer flow</td>
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<td>20 °C summer flow (with your E)</td>
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<tr>
<td>20 °C summer flow + nonpoint source</td>
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<tr>
<td>20 °C summer flow + nonpoint source (with your E)</td>
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</tbody>
</table>

* The check indicates that questions are asked in the text of the problem set. Please provide answers to all questions. E indicates that you should also be reporting an adjusted efficiency.