STELLA Assignment #4 - Photosynthesis and Respiration

To get a better picture of how dissolved oxygen is affected by a stream’s characteristics, it is necessary to include additional sources and sinks of oxygen. The source to be considered in this assignment is photosynthesis; the sink is respiration. Plant life in a stream, whether it is duckweed or a more common concern, such as algae, produces oxygen during the day (photosynthesis), some of which is used for respiration. At night, oxygen is consumed for respiration in the absence of production through photosynthesis. To more accurately describe the change in oxygen level, it is necessary to incorporate these effects into your STELLA dissolved oxygen model.

1) Let’s try a respiration amount of 1.8 mg/L*day. This indicates that throughout a 24 hour period the amount of oxygen used for respiration is 1.8 mg/L. To apply this to STELLA, it is only required to add this flow; no other adjustments are needed. [Note: start with the model from part 3 of STELLA assignment #3, which includes the non-point source.] Remember, this is a form of oxygen depletion; hence, the flow begins in the D.O. stock and "flows out".

Now for something a bit more challenging!

2) For modeling photosynthesis, it may be helpful to refer to the following reference (excerpts from the article are included in your course notes): "Diurnal water quality modeling - a case study" by Deb and Bowers, Journal WPCF, 55(12): 1476 (1983).

This article evaluates the DIURNAL model developed by O'Connor and DiToro at Manhattan College. The model uses the following equation for modeling photosynthetic oxygen production:

\[ P(t) = p_m \left( \frac{2p}{\pi} + \sum_{n=1}^{\infty} b_n \cos \left( \frac{2\pi n}{\rho} \left( t - t_s - \frac{\rho}{2} \right) \right) \right) \]

where:

- \( P(t) \) is algal photosynthetic oxygen production
- \( p_m \) is maximum rate of photosynthetic oxygen production (in mg/L*day)
- \( t_s \) is time at which the source (i.e., sun rise) begins (in days)
- \( \rho \) is the fraction of the 24 hour day over which the sun is active.

and:

\[ b_n = \cos(n\pi\rho) \left[ \frac{4\pi / \rho}{\left( \frac{\pi}{\rho} \right)^2 - (2\pi n)^2} \right] \]

This equation for photosynthetic oxygen production is a Fourier series. Ideally, one would sum all the expressions from \( n=1 \) to \( \infty \), to obtain the complete equation for \( P(t) \). STELLA does not have the capabilities to solve a summation, hence it would be up to you to expand the equation.
using n=1, then add the expression for n = 2, then n = 3, and so on. To properly use this series, it would take you a very long time to enter in an infinite number of expressions. Luckily, one often finds that it is only necessary to expand the series through the first few terms. The first three terms will suffice for this problem.

Photosynthesis will act as a source, and should flow into the DO reservoir in your model. The flow of O$_2$ will be a function of $b_1$, $b_2$, $b_3$, $\rho$, $t_s$, and $p_m$. You will need converters for each of these terms designated with appropriate labels. The converter for the fraction of time the source is active ($\rho$) will also be an input to the $b_1$, $b_2$, $b_3$ terms. [Note: be careful entering your equations, it’s easy to misplace a parenthesis.] Let’s run the model with photosynthesis and respiration under the following winter conditions:

- $T = 10^\circ C$ (remember this affects the saturation D.O. level; at $10^\circ C$, $C^* = 11.28$ mg/L)
- D.O. in stream (before the waste discharge input) = 10 mg/L
- D.O. level in wastewater = 1 mg/L
- Treatment Plant Efficiency = 0.80
- Non-point Source BOD input rate = $5.0 \times 10^5$ mg/km*day
- Wastewater BOD = 250 mg/L
- Stream BOD (before the waste discharge input) = 1.0 mg/L
- Plant flow rate = $5 \times 10^5$ L/day
- Stream flow rate = $5 \times 10^6$ L/day
- Stream velocity = 5 km/day
- $k_1$ (temperature dependent rate constant for BOD use) = 0.23 day$^{-1}$ at $20^\circ C$.
- $\theta_1$ (constant used for adjusting $k_1$ for temperature) = 1.047
- $k_2$ (rate constant for reaeration) at $20^\circ C$ is 0.28 day$^{-1}$
- $\theta_2$ (constant used for adjusting $k_2$ for temperature) = 1.025
- $k_3$ (rate constant for BOD loss to sediments) = 0.04/day
- $t_s$ = 0.20 days
- $\rho$ = 0.51 (good old sunny Ithaca)
- $p_m$ = 4 mg/L*day
- Model time step (DT) = 0.01 day
- Simulation time interval = 30 days (this should be long enough for you to arrive at a pseudo-steady state with stable daily oscillations of D.O.)

Under these conditions, what is the minimum D.O. in the stream? Roughly what is the daily magnitude of D.O. oscillation caused by photosynthesis and respiration? As usual, please turn in print outs of your STELLA diagram, equations, a graph of D.O. and BOD and a table of D.O. and BOD vs. distance (please use a report interval of 0.2 days). Be sure to clearly indicate your solutions to the questions.