1) A nozzle directs a stream of water into a vane that splits the stream in two. Each of the resulting streams leaves the vane at an angle of 45°. The velocity of the water leaving the nozzle is 20 m/s. The diameter of the nozzle is 3 cm. How much force is required to keep the vane from moving?

\[
\begin{align*}
\rho & := 1000 \text{ kg/m}^3 \\
d_{\text{nozzle}} & := 3 \text{ cm} \\
A_{\text{nozzle}} & := \frac{\pi \cdot d_{\text{nozzle}}^2}{4} \\
Q & := A_{\text{nozzle}} \cdot V_{1x} \\
V_{1x} & := 20 \frac{\text{m}}{\text{s}} \\
V_{2x} & := -V_{1x} \cdot (\sin(45\text{deg})) \\
M_{1x} & := -\rho \cdot Q \cdot V_{1x} \\
M_{2x} & := \rho \cdot Q \cdot V_{2x} \\
F_{ssx} & := M_{1x} + M_{2x} - W - F_{p1} - F_{p2} \\
F_{ssx} & = -482.673 \text{ N}
\end{align*}
\]
2) (25 points) Wind blows over a shallow (infinitely long and wide) tank of water such that a stagnation tube inserted at ½ depth measures a water height \( h \). Derive an equation for the shear at the free surface as a function of \( h \).

Use Bernoulli between a point upstream of the stagnation tube and the top of the stagnation tube. Elevation datum is surface of water.

\[
\frac{p_1}{\rho g} + z_1 + \frac{u^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{v_2^2}{2g}
\]

\[
\frac{u^2}{2g} = h \quad \text{at } a/2
\]

\[
\rho a = -\rho g - \nabla p + \mu \nabla^2 V
\]

General equation describing laminar flow between parallel plates where \( x \) is in direction of flow and there is no acceleration

\[
\frac{d}{dx} p = 0 \quad g_x = 0
\]

\[
\mu \left( \frac{d^2 u}{dy^2} \right) = 0
\]

\[
\int \mu \left( \frac{d^2 u}{dy^2} \right) dy = A
\]

\[
\mu \left( \frac{d u}{dy} \right) = A
\]

note that shear = \( A \! \)!

no shear boundary condition
\[ \int \mu \left( \frac{d}{dy} u \right) dy = \int A \, dy \]

\[ \mu \cdot u = Ay + B \]

no slip boundary condition
\[ u := 0 \quad \text{at} \quad y := 0 \]
\[ 0 = 0 + B \quad \text{B} = 0 \]
\[ \mu \cdot u = Ay \]

known velocity at a/2 boundary condition
\[ u := \sqrt{2 \cdot g \cdot h} \quad \text{at} \quad y := \frac{a}{2} \]
\[ \mu \sqrt{2 \cdot g \cdot h} = \frac{A \cdot a}{2} \quad A := \frac{2 \mu \sqrt{2 \cdot g \cdot h}}{a} \]
\[ \tau_{fs} := \frac{2 \mu \sqrt{2 \cdot g \cdot h}}{a} \]
3) (30 points) A 0.5-mm inside diameter capillary tube, inclined 20° from the horizontal, carries 20°C water. The flow is laminar.

A) (15 points) What is the flow rate (including direction of flow) in the capillary tube in mL/min?

B) (5 points) What is the maximum water velocity (centerline velocity)?

C) (10 points) What is the total shear force of the capillary tube on the water in the 2 m section?

Here I am defining x positive in the direction of flow (from right to left)

\( \Delta p := 20 \text{kPa} - 40 \text{kPa} \)
\( \Delta p = -2 \times 10^4 \text{Pa} \)
\( \Delta x := 2 \text{m} \)
\( g_x := g \cdot \cos(70\degree) \quad g_x = 3.354 \text{ m/s}^2 \quad \mu := 0.001 \text{ N·s/m}^2 \)
\( R := 0.25 \text{mm} \)

\( Q := \frac{-\pi R^4}{8\mu} \left( \rho \cdot g_x + \frac{\Delta p}{\Delta x} \right) \)

\( Q = 1.019 \times 10^{-5} \text{ L/s} \quad Q = 0.612 \text{ mL/min} \quad h_l = 1.355 \text{ m} \)

Flow is from right to left

\( V_{\text{ave}} := \frac{Q}{\pi R^2} \quad V_{\text{ave}} = 0.052 \text{ m/s} \) toward the left!

\( V_{\text{center}} := 2 \cdot V_{\text{ave}} \quad V_{\text{center}} = 0.104 \text{ m/s} \) for laminar flow (parabolic velocity distribution)

\( \text{Re} := \frac{V_{\text{ave}} \cdot R \cdot \rho \cdot \mu}{\mu} \quad \text{Re} = 25.961 \)

\( \Delta z := \frac{g_x}{g} \cdot \Delta x \)

\( \tau_0 := \frac{R}{2} \left( \rho \cdot g_x + \frac{\Delta p}{\Delta x} \right) \)

\( \tau_0 = -0.831 \text{ Pa} \)

\( F_{\text{shear}} := \tau_0 \cdot \Delta x \cdot 2 \cdot \pi \cdot R \)

\( F_{\text{shear}} = -2.61 \times 10^{-3} \text{ N} \)

\( h_l := \frac{\rho \cdot g \cdot h_l \cdot R \cdot 2}{4 \Delta x} \)

\( \tau_0 := -0.831 \text{ Pa} \)
4) (15 points) How fast will this sprinkler rotate given a flow rate of 4 L/s, a nozzle diameter of 2 cm, a sprinkler radius of 40 cm and a torque of 1.5 N-m?

\[
\begin{align*}
\text{d} & := 2\text{cm} \\
\text{rsprinkler} & := 40\text{cm} \\
T_Z & := 1.5\text{N\cdot m} \\
Q & := 4 \frac{\text{L}}{s} \\
V_t & := \omega \cdot \text{rsprinkler} \\
\omega & := \frac{T_Z}{(\rho \cdot Q \cdot r_{\text{sprinkler}})^2} \\
\omega & = 2.344 \text{ Hz}
\end{align*}
\]
\[ \rho g \frac{h_1}{l} = - \left( \frac{dp}{dx} + \rho g_x \right) \]

\[ z_2 := 2m \cdot \sin \left( \frac{20 \cdot \pi}{360} \right) \]

\[ z_2 = 0.684 \text{ m} \]

\[ p_1 := 40 \text{kPa} \]
\[ p_2 := 20 \text{kPa} \]

\[ h_1 := \frac{p_1 - p_2}{\rho g} - z_2 \]

\[ h_1 = 1.355 \text{ m} \]