Homework 10

Problem 1
The total surface area of a Toyota Matrix is approximately 30 m² (including top and bottom) and the length is approximately 4.5 m.
What would the coefficient of drag be for a Toyota Matrix traveling at 65 mph (30 m/s) if the only drag were viscous?

\[ A_{\text{surface}} := 30 \text{ m}^2 \]

\[ \nu_{\text{air}} := 14.6 \times 10^{-6} \text{ m}^2 / \text{s} \]

\[ L := 4.5 \text{ m} \]

\[ \rho_{\text{air}} := 1.22 \text{ kg/m}^3 \]

U := 30 \text{ m/s}

Solution 1

\[ \text{Re} = \frac{UL}{\nu_{\text{air}}} = 9.247 \times 10^6 \]

\[ C_d := 0.003 \]

\[ F_d := \frac{C_d \rho_{\text{air}} U^2 A_{\text{surface}}}{2} \]

\[ F_d = 49.41 \text{ N} \]

The viscous drag is tiny compared with the pressure drag.

Problem 2
Water enters a smooth pipe such that the initial velocity profile is uniform. In this problem you will model the entrance region of the pipe where the boundary layer is developing as if the pipe were a flat plate. Then you will compare the flat plate model with the shear in the pipe based on the pipe flow equations that are based on uniform flow. Note that the flat plate approximation is good as long the boundary layer thickness is small compared to the pipe radius.

D := 1 m

\[ U := 1 \text{ m/s} \]

\[ \nu := 1 \times 10^{-6} \text{ m}^2 / \text{s} \]

\[ \rho := 1000 \text{ kg/m}^3 \]

A) Estimate the length of the laminar boundary layer
The boundary layer becomes turbulent at 500,000 which happens at

\[ x_{\text{transition}} := \frac{\text{Re}_{\text{transition}} \nu}{U} \]

\[ x_{\text{transition}} = 0.5 \text{ m} \]

transition between laminar and turbulent boundary layer

B) The flat plate approximation is reasonable up to the distance where the boundary layer grows to 10% of the radius. At what distance into the pipe does this occur?

\[ x := x_{\text{transition}} \cdot x_{\text{transition}} + 0.01 \text{m} . 100 \text{m} \]

\[ \delta(x) := 0.37 \cdot x \left( \frac{\nu}{U} \right)^{\frac{5}{4}} \]

equation for turbulent boundary layer thickness

As the boundary layer grows larger it becomes large relative to the radius and the flat plate in an unbounded medium is no longer a good approximation.

\[ \delta_{\text{max}} := \frac{D}{2} \cdot 0.1 \]

\[ x_{\text{max}} := \left[ \frac{\delta_{\text{max}} \left( \frac{U}{\nu} \right)^{\frac{1}{5}}}{0.37} \right]^4 \]

\[ x_{\text{max}} = 2.6 \text{ m} \]

Distance into the pipe where the boundary layer is 10% of the radius.

C) Plot the shear as a function of distance into the pipe in the turbulent region up to where the boundary layer thickness is 10% of the radius

\[ x := x_{\text{transition}} \cdot x_{\text{transition}} + 0.01 \text{m} . x_{\text{max}} \]

\[ \text{Re}_x(x) := \frac{U \cdot x}{\nu} \]
\[ \tau_{\text{turbulent}}(x) := 0.029 \cdot \rho \cdot U^2 \left( \frac{v}{U \cdot x} \right)^{\frac{1}{5}} \]

valid for \( 5 \times 10^5 < \text{Re}_l < 10^7 \)

What is the shear in the pipe after uniform flow is established as predicted by the pipe flow equations?

Pipe flow equations

\[ \text{Re} := \frac{U \cdot D}{\nu} \quad \text{Re} = 1 \times 10^6 \quad \varepsilon := 0 \text{mm} \]

\[ f := \frac{0.25}{\log \left( \frac{\varepsilon}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right)^2} \quad f = 0.012 \]

\[ h_f := \frac{f \cdot L \cdot U^2}{D \cdot 2 \cdot g} \]

\[ \tau_{0f} := \frac{\rho \cdot g \cdot h_f D}{4 \cdot L} \quad \text{relationship between shear and head loss in pipes (ignoring the sign which simply means that the shear is in the opposite direction of the velocity).} \]

\[ \tau_{0f} := \frac{\rho \cdot f \cdot U^2}{8} \quad \text{substitute Darcy Weisbach into equation for shear} \]
This is the shear in uniform pipe flow for a smooth pipe

Just for fun, I plot the shear in the laminar region and put it all on one graph

\[ \tau_{0f} = 1.451 \text{ Pa} \]

\[ \mu := \rho \cdot \nu \]

\[ \tau_{0\text{laminar}}(x) := 0.332 \sqrt{\frac{\mu \cdot \rho \cdot U^3}{x}} \]

\[ x := 0 \text{ m, } 1 \text{ cm} \ldots x_{\text{max}} \]

\[ \tau_{0\text{laminar}}(x_{\text{transition}}) = 0.47 \text{ Pa} \]

shear in the laminar boundary layer right before transition to turbulence. Note that this shear is significantly less than the shear later in the pipe!

\[ \tau_{0\text{plate}}(x) := \begin{cases} \tau_{0\text{laminar}}(x) & \text{if } x < x_{\text{transition}} \\ \tau_{0\text{turbulent}}(x) & \text{otherwise} \end{cases} \]

function for both laminar and turbulent shear

Note that with different velocities and diameters the results can be quite different. For example, it is possible to not have the transition to turbulence until the boundary layer has grown to fill the entire pipe. Also note that the shear in the turbulent entrance region was not much greater than that predicted from the pipe flow equations. The flat plate equations will predict a slightly lower shear far into the pipe because we aren't accounting for the fact that the velocity at the center of the pipe has to increase as the boundary layer slows down.

\[ \begin{array}{c}
\tau_{0\text{plate}}(x) \\
\tau_{0f}
\end{array} \]

\[ x \]

Problem 3

The Colombia Basin Irrigation Project Feeder Canal is 2.9 km in length, 7.5 m deep and 24 m wide at the base. One of the sidewalls has a slope of 1:1 and the other sidewall is vertical. It has the capacity to carry 450 m$^3$/s. The Manning $n$ for the canal is 0.012. You know the bottom of the channel is probably horizontal, but for this problem assume that the channel bottom and the water surface have the same slope.

A) What is the slope of the canal?

\[ L_{\text{channel}} := 2900 \text{ m} \]

\[ w_{\text{bottom}} := 24 \text{ m} \]

\[ h := 7.5 \text{ m} \]

\[ n := 0.012 \]

\[ \gamma := 9806 \frac{\text{N}}{\text{m}^3} \]

Manning equation (SI units)

\[ Q = \frac{1}{n} \cdot A_{\text{cs}} \cdot R_h \cdot \frac{2}{3} \cdot S_0 \]

\[ Q := 450 \frac{\text{m}^3}{\text{s}} \]

\[ A_{\text{cs}} := \left( w_{\text{bottom}} + \frac{h}{2} \right) \cdot h \]

\[ A_{\text{cs}} = 208.125 \text{ m}^2 \]

\[ P := h + w_{\text{bottom}} + \sqrt{2} \cdot h \]

\[ P = 42.107 \text{ m} \]
What is the total drop in the water surface elevation from one end of the canal to the other end?

The total drop is simply the slope times the length

$$\Delta z := S_0 \cdot L_{\text{channel}}$$

$$\Delta z = 0.232 \text{ m}$$

What is the average velocity in the canal?

$$V_{\text{channel}} := \frac{Q}{A_{cs}}$$

$$V_{\text{channel}} = 2.162 \text{ m/s}$$

What is the power loss over the length of the feeder canal?

$$h_l := S_0 \cdot L_{\text{channel}}$$

This is the same as the elevation drop of the surface for uniform flow

$$P_{\text{loss}} := \gamma \cdot Q \cdot h_l$$

$$P_{\text{loss}} = 1.023 \text{ MW}$$

Small compared with the power required to lift the water up to the canal from Lake Mead.

Problem 5

Water flows in a finished concrete pipe that is completely full and the pressure is constant all along the pipe. If the pipe slope is as given, determine the flow rate by using both open-channel flow methods and pipe flow methods

$$n := 0.012 \frac{s}{\sqrt{m}}$$

$$\varepsilon := 1 \text{ mm}$$

$$D := 2 \text{ m}$$

$$S_0 := 0.005$$
Open channel flow equations

\[ P := \pi \cdot D \]

\[ A := \frac{\pi \cdot D^2}{4} \]

\[ R_h := \frac{A}{P} \quad R_h = 0.5 \text{ m} \]

\[ Q_{\text{manning}} := \frac{1}{n} \cdot A \cdot R_h^{\frac{2}{3}} \cdot S_0^\frac{1}{2} \]

\[ v = 1 \times 10^{-6} \text{ m}^2 \text{ s}^{-1} \]

Pipe flow equations

\[ Q_{\text{pipe}} := \frac{-\pi}{\sqrt{2}} \cdot D^{2.5} \cdot \sqrt{\frac{g \cdot h_f}{L}} \cdot \log \left( \frac{\frac{e}{3.7D} + 2.51v \cdot \sqrt{\frac{L}{2g \cdot h_f \cdot D^3}}} {\frac{1}{2g \cdot S_0 \cdot D^3}} \right) \]

\[ S_0 = \frac{h_f}{L} \]

Substitute \( S_0 = \frac{h_f}{L} \)

\[ Q_{\text{manning}} = 11.662 \text{ m}^3 \text{ s}^{-1} \]

\[ Q_{\text{pipe}} = 10.742 \text{ m}^3 \text{ s}^{-1} \]