Homework 5

Make sure you clearly draw and label the control surfaces and the coordinate system.

**Problem 1**

The bend is in the horizontal plane. You may ignore the vertical forces. Mechanical energy losses through the bend are negligible.

A) Draw a control volume and clearly label the control surfaces.

B) Calculate the magnitude of the resultant force required to keep the bend in place.

C) Calculate the direction of the resultant force required to keep the bend in place.

**Solution 1**

\[ p_1 := p_{gage} \]

Use continuity to find the velocities at each control surface

\[ V := \frac{4Q}{\pi d^2} \quad V = 2.829 \text{ m/s} \]

To find the force we will need to know \( p_2 \). Use Bernoulli equation

\[
\frac{p_1}{\rho \cdot g} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho \cdot g} + z_2 + \frac{V_2^2}{2g}
\]

Therefore

\[ p_2 := p_1 \quad p_2 = 75 \text{ kPa} \]

Now find the values for each of the terms in the momentum equation

\[ F_{p1x} := \frac{\pi d^2}{4} p_1 \quad F_{p1x} = 132.5 \text{ kN} \]

\[ F_{p2x} := \frac{-\pi d^2}{4} p_2 \cdot \cos(\theta) \quad F_{p2x} = -114.8 \text{ kN} \]

\[ M_{1x} := -\rho \cdot Q \cdot V \quad M_{1x} = -14.1 \text{ kN} \]

\[ M_{2x} := \rho \cdot Q \cdot V \cdot \cos(\theta) \quad M_{2x} = 12.3 \text{ kN} \]

\[ F_{ssx} := M_{1x} + M_{2x} - (F_{p1x} + F_{p2x}) \quad F_{ssx} = -19.7 \text{ kN} \]

\[ F_{p1y} := 0 \quad F_{p1y} = 0 \]
\[
F_{p2y} = \frac{\pi d^2}{4} \cdot p_2 \cdot \sin(\theta) \quad F_{p2y} = 66.3 \text{ kN}
\]

\[
M_{1y} = 0 \text{ kN}
\]

\[
M_{2y} = -\rho \cdot Q \cdot V \cdot \sin(\theta) \quad M_{2y} = -7.1 \text{ kN}
\]

\[
F_{ssy} := M_{1y} + M_{2y} - (F_{p1y} + F_{p2y}) \quad F_{ssy} = -73.3 \text{ kN}
\]

\[
F_{ss} := \left( F_{ssx}^2 + F_{ssy}^2 \right)^{0.5} \quad F_{ss} = 75.9 \text{ kN}
\]

\[
\theta_{\text{force}} := \text{angle}(F_{ssx}, F_{ssy}) \quad \theta_{\text{force}} = 255 \text{ deg}
\]

measured from the positive x axis. Note that this force bisects the angle created by the pipe.

\[
F_{ss} = 75.9 \text{ kN}
\]

**Problem 2**

A nozzle directs a stream of water into a vane that splits the stream in two. Each of the resulting streams leaves the vane at the same angle. The vane is moving away from the nozzle. Calculate the force acting on the vane and the power of the vane. Draw the control volume carefully and define the velocities and flows very carefully!

\[
\theta := 30 \text{ deg} \\
d_{\text{nozzle}} := 3 \text{ cm} \\
V_{\text{vane}} := 15 \frac{\text{m}}{\text{s}} \\
V_{\text{nozzle}} := 20 \frac{\text{m}}{\text{s}} \\
\rho := 1000 \frac{\text{kg}}{\text{m}^3}
\]

\[
\text{Solution 2}
\]

The control volume is traveling with the vane toward the right. This means that the flow rate into the control volume is NOT the same as the flow rate out of the nozzle! There are 3 control surfaces, but the two leaving the control volume are similar and can be treated as a single control surface.

\[
W := 0 \text{ N} \\
F_{p1} := 0 \text{ N} \\
F_{p2} := 0 \text{ N} \\
A_{\text{nozzle}} := \frac{\pi \cdot d_{\text{nozzle}}^2}{4} \\
A_{\text{nozzle}} = 7.069 \times 10^{-4} \text{ m}^2
\]
V₁x := V_{nozzle} - V_{vane} \quad V₁x = 5 \text{ m/s} \quad Q := A_{nozzle} V₁x \quad Q = 3.53 \times 10^{-3} \text{ m}^3/\text{s}

V₂x := -V₁x \cdot \cos(\theta) \quad V₂x = -4.33 \text{ m/s}

M₁x := -\rho \cdot Q \cdot V₁x

M₂x := \rho \cdot Q \cdot V₂x

F_{ssx} := M₁x + M₂x - W - F_{p1} - F_{p2}

F_{ssx} = -33 \text{ N} \quad \text{This is the force acting on the water. The force acting on the vane is in the opposite direction.}

F_{vane} := -F_{ssx} \quad F_{vane} = 33 \text{ N}

The power is simple the force times the velocity.

P := F_{vane} \cdot V_{vane}

P = 495 \text{ W}

**Problem 3**

A volumetric detector is used to measure flow rates. For each part of the question indicate whether you used a control volume or point to point equation and label control surfaces or points that you use to obtain your solution.

A) What is the velocity of the jet at the water surface in the volumetric detector?

B) What is the diameter of the jet when it reaches the water surface in the volumetric detector (as shown in the drawing)?

C) A stagnation tube connected to a pressure transducer is directed into the falling jet at the level of the water in the volumetric detector. What pressure should the pressure transducer report?

D) The balance measures force but is calibrated to display liters of water. The balance displays zero when there is no water in the container and the jet is turned off. What will the balance display when the water level is as shown in the drawing?
Solution 3

A point to point equation (Bernoulli)

\[
\frac{p_{\text{jet}}}{\rho g} + \frac{V_{\text{jet}}^2}{2g} + z_{\text{jet}} = \frac{p_{\text{impact}}}{\rho g} + \frac{V_{\text{impact}}^2}{2g} + z_{\text{impact}}
\]

\[
\frac{V_{\text{jet}}^2}{2g} + z_{\text{jet}} = \frac{V_{\text{impact}}^2}{2g}
\]

\[
V_{\text{impact}} := \left[ 2g \left( \frac{V_{\text{jet}}^2}{2g} + z_{\text{jet}} \right) \right]^{0.5}
\]

\[
V_{\text{impact}} = 5.812 \text{ m/s}
\]

B The flow is steady and thus \(V \cdot A\) is constant.

\[
d_{\text{impact}} := \left( \frac{4Q}{\pi V_{\text{impact}}} \right)^{0.5}
\]

\[
d_{\text{impact}} = 4.68 \text{ mm}
\]
C Find the pressure at the impact location as would be measured by a stagnation tube using point to point equ:

(Bernoulli)

\[
\frac{p_{\text{jet}}}{\rho \cdot g} + \frac{V_{\text{jet}}^2}{2g} + z_{\text{jet}} = \frac{p_{\text{impact}}}{\rho \cdot g} + \frac{V_{\text{impact}}^2}{2g} + z_{\text{impact}}
\]

\[
\frac{V_{\text{jet}}^2}{2g} + z_{\text{jet}} = \frac{p_{\text{impact}}}{\rho \cdot g}
\]

\[p_{\text{impact}} = \frac{1}{2} \left( V_{\text{jet}}^2 + 2 \cdot z_{\text{jet}} \cdot g \right) \cdot \rho\]

\[p_{\text{impact}} = 17 \text{kPa}\]

D Find the force acting on the balance using a control volume equation (Linear Momentum). At the instant in

when the water level is as drawn the control volume includes all of the water in the tank. Water is entering the

volume at the point of impact (cs1) and water is leaving the control volume through all of the remaining free s

(cs2). The pressure at cs1 is approximately zero because the streamlines of the jet are very close to parallel (al

they do diverge quickly once they pass the free surface.

\[F_{p2} = 0 \text{N} \quad F_{p1} = 0 \text{N}\]

\[d_{vd} := 10 \text{cm} \quad h_{vd} := 15 \text{cm}
\]  

\[vd \text{ is volumetric detector}\]

\[M_1 := -\rho \cdot Q \cdot \left(-V_{\text{impact}}\right) \quad M_1 = 0.581 \text{ N}\]

\[V_{\text{up}} := V_{\text{impact}} \cdot \frac{d_{\text{impact}}^2}{d_{vd}^2} \quad V_{\text{up}} = 0.013 \text{ m/s}
\]

\[M_2 := \rho \cdot Q \cdot V_{\text{up}} \quad M_2 = 1.273 \times 10^{-3} \text{ N}\]

\[\text{I am neglecting the tiny area taken up by the incoming jet.}\]

\[\text{This momentum term is negligible.}\]

\[\text{Volume}_{\text{actual}} := \frac{\pi}{4} \left( d_{vd}^2 \right) h_{vd} \quad \text{Volume}_{\text{actual}} = 1.178 \text{ L}\]

\[\text{Weight} := -\text{Volume}_{\text{actual}} \cdot \rho \cdot g \quad \text{Weight} = -11.553 \text{ N}\]

\[F_{ss} := M_1 + M_2 - \left( \text{Weight} + F_{p1} + F_{p2} \right) \quad F_{ss} = 12.136 \text{ N}\]

\[\text{The volume measured is slightly larger than the actual volume. The error is due to the momentum of the incoming jet at the point of impact.}\]

\[\text{Volume}_{\text{measured}} = 1.237 \text{ L}\]
**Problem 4**

Air flows from a pipe and causes the water in a stagnation tube to move as shown. The air then strikes a fixed vane. What is the horizontal component of the force that the air jet exerts on the vane?

**Solution 4**

\[
\rho_{\text{air}} := 1.22 \, \text{kg/m}^3
\]

\[
\rho_{\text{water}} := 1000 \, \text{kg/m}^3
\]

Use the Bernoulli equation to find the velocity of the jet. Point 1 is at the nozzle exit and point 2 is at the stagnation tube.

\[
\frac{p_1}{\rho \cdot g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho \cdot g} + \frac{V_2^2}{2g} + z_2
\]

\[
z_1 = z_2
\]

\[
p_2 := p_{\text{jet stagnation}}
\]

\[
\frac{V_1^2}{2g} = \frac{p_2}{\rho_{\text{air}} \cdot g}
\]

\[
V_1 := \sqrt{\frac{2 \cdot p_2}{\rho_{\text{air}}}}
\]

\[
V_1 = 49.107 \, \text{m/s}
\]

\[
Q := V_1 \cdot \frac{\pi \cdot d^2}{4}
\]

\[
Q = 0.096 \, \text{m}^3/\text{s}
\]

\[
M_{1x} := -\rho_{\text{air}} \cdot Q \cdot V_1
\]

\[
M_{1x} = -6 \, \text{N}
\]

\[
M_{2x} := -\rho_{\text{air}} \cdot Q \cdot V_1 \cos(\theta)
\]

\[
M_{2x} = -3 \, \text{N}
\]

\[
F_{\text{ssx}} := M_{1x} + M_{2x}
\]

\[
F_{\text{ssx}} = -8.66 \, \text{N}
\]

This is the force of the vane on the fluid. The fluid pushes the vane toward the right.

\[
F_{\text{ssx}} = -8.66 \, \text{N}
\]