Homework 9

Problem 1
What diameter of pipe is required to transport water horizontally given the following conditions?

\[ Q_{\text{max}} := 6 \frac{L}{s} \quad \text{maximum design flow} \]

\[ Q_{\text{min}} := 3 \frac{L}{s} \quad \text{minimum design flow} \]

\[ \Delta p_{\text{max}} := -200 \text{kPa} \quad \text{maximum pressure drop} \]

\[ L := 500 \text{m} \quad \text{Length of the pipe} \]

\[ \varepsilon := 0.12 \text{mm} \quad \text{Pipe roughness} \]

Solution 1

\[ \rho := 1000 \frac{\text{kg}}{\text{m}^3} \]

\[ \nu := 1.007 \times 10^{-6} \frac{\text{m}^2}{\text{s}} \]

\[ \alpha_1 \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 + H_p = \alpha_2 \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + H_t + h_l \quad \text{uniform velocity} \]

\[ \frac{p_1}{\gamma} = \frac{p_2}{\gamma} + h_l \quad \text{constant elevation} \]

all head loss is due to viscous (major) losses

\[ h_f := \frac{-\Delta p_{\text{max}}}{\rho \cdot g} \quad h_f = 20.394 \text{ m} \]

\[ Q := Q_{\text{max}} \]

\[ D := 0.66 \left[ \left( \frac{\varepsilon}{1.25} \right) \left( \frac{L \cdot Q^2}{g \cdot h_f} \right)^{4.75} + \nu \cdot Q^{0.4} \left( \frac{L}{g \cdot h_f} \right)^{5.2} \right]^{0.04} \]

\[ D = 7.211 \text{ cm} \]

Problem 2 (continued from 1)
What is the wall shear stress for the pipe in problem 1?

Solution 2
Problem 3
This problem builds on analysis that you completed in HW 2. Repeated information is given in red. I install radiant floor heating system in my friend's house. The house stayed warm all winter even though the circulator pump that is supposed to pump the water through the system never turned on. Determine the pressure at point A in the hot water supply and at point B in the cold water return assuming that the pressure at the bottom of the water heater where the cold water enters and that as soon as the water enters the water heater it is heated. Also calculate the pressure difference. Note that points A and B are at the same elevation a distance $h$ above the location where the cold water enters the water heater.

A) Estimate the flow of water through a single pipe loop (there are 6 loops in parallel to heat the first floor).
B) Given the heat capacity of water estimate the total rate of heat transfer for the 6 pipe loops.

\[
\begin{align*}
\tau_0 & := \frac{\rho g h_f D}{4 L} \\
\tau_0 & = 7.211 \text{ Pa}
\end{align*}
\]

This similarity of numbers is pure coincidence.
The kinematic viscosity will be a function of location in the pipe. To simplify the problem you may assume that the kinematic viscosity is constant with a value from the average temperature. Assume the viscosity is as given:

\[ \nu := 5.5 \times 10^{-7} \text{ m}^2/\text{s} \]

**Solution 3**

The pressure at A is higher than the pressure at B. This difference in pressure causes the water in the pipe to flow in the direction of the arrows and thus hot water was continuously carried up into the basement ceiling.

Now apply the energy equation between control surfaces at A (surface 1) and B (surface 2).

\[
\alpha_1 \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 + H_p = \alpha_2 \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + H_t + h_f
\]

Uniform velocity
pump is off
no turbine

\[ \Delta p := p_B - p_A \]
\[ \Delta p = -351.078 \text{ Pa} \]

\[ h_f := \frac{-\Delta p}{\rho g} \]
\[ h_f = 35.8 \text{ mm} \]

Assume turbulent
\[ D := d_{\text{tube}} \]
\[ L := L_{\text{tube}} \]

\[
Q := \frac{-\pi}{\sqrt{2}} D^{2.5} \sqrt{\frac{g \cdot h_f}{L}} \log \left( \frac{e}{3.7D} + 2.51\nu \cdot \sqrt{\frac{L}{2g \cdot h_f \cdot D^3}} \right)
\]

\[ \nu = 5.5 \times 10^{-7} \text{ m}^2/\text{s} \]
\[ Q = 0.28 \text{ l/min} \]

\[ \text{Re} := \frac{4 \cdot Q}{\pi \cdot D \cdot \nu} \quad \text{Re} = 851.785 \]

I was unlucky! Flow is laminar so use Hagen-Poiseuille

\[ Q := \frac{\pi \cdot D^4 \cdot g \cdot h_f}{128 \cdot \nu \cdot L} \]

Now calculate the amount of heat transferred.

\[ \text{Heat} := Q \cdot \rho \cdot c_p \cdot (T_{\text{hot}} - T_{\text{cold}}) \]

Heat = 0.746 kW

With 6 loops the amount of heat transferred is

\[ \text{Heat}_{\text{total}} := 6 \cdot \text{Heat} \]

\[ Q = 0.267 \text{ l/min} \]

Heat_{\text{total}} = 4.476 kW

This is a lot of heat! This is enough to heat the house except on the very coldest days.

**Problem 4**

A pump is used to raise water from a lake to a storage tank. The pipes are ductile iron \( \varepsilon := 0.045 \text{ mm} \)

A) Draw a control volume and clearly label the control surfaces.

B) What is the head loss in the pipeline if you neglect minor losses and neglect the kinetic energy term?
B) \[
\alpha_1 \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 + H_p = \alpha_2 \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + H_t + h_L
\]

\[
\frac{p_1}{\gamma} - z_1 = 0 \quad \alpha_1 := 0 \quad \alpha_2 := 0 \quad H_t := 0
\]

elevation datum is lake surface

\[z_2 := h_2 + h_1 \quad p_2 := 0 Pa\]

\[H_p = \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L\] We don't know the velocity, so we will neglect the kinetic energy term to get an estimate of the flow rate. We will then correct this in step E.

\[h_L := H_p - z_2\]

\[h_L = 24 m\]

C) Now we solve for the flow (neglecting minor losses and thus all losses are due to shear).

\[h_f := h_L \quad h_f = 24 m \quad D := D_1 \quad L := L_1 + L_2 \quad L = 900 m\]

\[Q := \frac{-\pi \cdot D^2}{\sqrt{2}} \cdot \sqrt{\frac{g \cdot h_f}{L} \cdot \log \left( \frac{e}{3.7D} + 2.51v \cdot \sqrt{\frac{L}{2g \cdot h_f \cdot D^3}} \right)} \quad v := 0.000001 \frac{m^2}{s}\]

\[Q = 0.077 \frac{m^3}{s}\] neglecting minor losses

D) What is the friction factor for this pipe? (for your benefit, find this point on the Moody diagram)
Re := \frac{4 Q}{\pi D \cdot \nu}

Re = 4.802 \times 10^5

f := \frac{0.25}{\log \left( \frac{\varepsilon}{3.7D} + \frac{5.74}{Re^{0.9}} \right)^2}

This is in the region where both roughness and Re matter. But it
doesn't change very much with small changes in the Re.

\[ f = 0.019 \]

E) What dimensionless quantities could you compare to determine what fraction of the total head loss is due to
major losses? The minor losses include the exit loss, entrance loss, and 6 elbows with a loss coefficient of 0.9.

Compare \( f \frac{L}{D} \) with \( f \frac{L}{D} + \sum K \)

Note that all of these terms are pressure coefficients.

\[ h_{\text{majorratio}} := \frac{f \frac{L}{D}}{f \frac{L}{D} + K_{\text{total}}} \]

K_{\text{exit}} := 1 \quad \text{The accounts for the kinetic energy}
leaving the control volume
K_{\text{entrance}} := 1
K_{\text{elbow}} := 0.9
K_{\text{total}} := 6K_{\text{elbow}} + K_{\text{entrance}} + K_{\text{exit}}
K_{\text{total}} = 7.4

\[ h_{\text{majorratio}} = 0.92 \quad \text{ratio of major losses to total head loss (because f is relatively constant!)} \]

F) Recalculate the flow through the pipeline by correcting your estimate of the major losses

\[ h_{f} := h_L \cdot h_{\text{majorratio}} \]

\[ h_{f} = 22.074 \text{ m} \]

\[ Q := -\frac{\pi}{\sqrt{2}} D^{2.5} \sqrt{\frac{g}{L} h_{f} \cdot \log \left( \frac{\varepsilon}{3.7D} + 2.51v \cdot \sqrt{\frac{L}{2g h_{f} D^3}} \right)} \]

\[ Q = 0.073 \text{ m}^3/\text{s} \]
Problem 5 OPTIONAL!

A surge chamber is used to prevent water hammer in a small pipeline. The pressure, \( p_1 \), is constant. There are no minor losses between the point where \( p_1 \) is measured and the pipeline. Find the height of water in the surge chamber when the angle valve is open. This problem includes several complexities, but you have the necessary skills to solve all of them. This problem can be solved many different ways with different levels of difficulty and different levels of precision. Your assignment is to first describe (in words) how you would solve this problem without making any simplifying assumptions. Then discuss possible simplifying assumptions. Use the simplifying assumptions, but then justify those assumptions. (The final air height is 0.46 m)

Solution steps without any simplification:
1) Assume the air temperature in the surge chamber is constant.
2) Find initial absolute pressure in the surge chamber using statics
3) find moles of air in surge chamber per unit cross sectional area of the surge chamber
4) find Q in the pipe with valve open
5) find headloss between \( P_1 \) and center of Tee
6) find pressure at base of Tee
7) find \( h_w \) given moles of air in tube and pressure at base of Tee using statics and \( PV=nRT \)

Simplifying assumptions

Minor losses are minor
Change in air pressure due to \( h_w \) is small

Then gage pressure in surge chamber is proportional to the lengths of the pipeline
The absolute pressure in surge chamber times the air height is constant (from \( PV \) is constant).
Defense of simplifications

**Are minor losses minor?**

\[
K_{\text{minor}} := K_v + K_{\text{tee}}
\]

\[
\text{headlossratio} := \frac{K_{\text{minor}}}{f \frac{L_1}{d_1} + K_{\text{minor}}}
\]

Assume \( f := 0.020 \)

\[
\text{headlossratio} = 0.015
\]

Therefore the minor losses are minor.

**Is the pressure change due to the water level small compared with the pressure change due to the change in head loss? Compare the two.**

The pressure change due to 0.5 m of water is about 5 kPa. The pressure change in the surge chamber is almost 400 kPa. Thus the volume error due to neglecting the change in water column height will be negligible.

**The solution below is without making simplifying assumptions**

\[
P_{\text{surge,absolute1}} := P_1 + P_{\text{atm}} - \rho g h_{w1}
\]

\[
P_{\text{surge,absolute1}} = 4.974 \times 10^5 \text{ Pa}
\]

from ideal gas law \( P*V \) is a constant and thus \( P*ha \) is a constant

\[
P_{\text{surge,absolute1}} h_{a1} = P_{\text{surge,absolute2}} h_{a2}
\]

Calculate flow using estimate of minor losses based on estimate of \( f \)

\[
\frac{P_1}{\rho g} h_L = 40.789 \text{ m}
\]

\[
L := L_1 + L_2 \quad \text{D} := d_1
\]

\[
h_f := h_L - \text{headlossratio} h_L \quad h_f = 40.16 \text{ m}
\]
\[
Q := \frac{-\pi}{\sqrt{2}} D^{2.5} \sqrt{\frac{g \cdot h_f}{L}} \cdot \log \left( \frac{\varepsilon}{3.7D} + 2.51v \cdot \sqrt{\frac{L}{2g \cdot h_f D^3}} \right)
\]

\[Q = 3.278 \times 10^{-3} \text{ m}^3/\text{s}\]

\[
\text{Re} := \frac{4 \cdot Q}{\pi \cdot D \cdot \nu}
\]

\[\text{Re} = 8.347 \times 10^4\]

\[
f' := \frac{0.25 \log \left( \frac{\varepsilon}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right)^2}{L_1 \cdot \frac{d_1}{\nu} + K_{\text{minor}}}\]

\[f = 0.028\]

So my initial guess was off a bit

\[
\text{headlossratio} := \frac{K_{\text{minor}}}{f' \cdot \frac{L_1}{d_1} + K_{\text{minor}}}
\]

\[\text{headlossratio} = 0.011\]

\[
h_f := h_L - \text{headlossratio} \cdot h_L
\]

\[h_f = 40.356 \text{ m}\]

\[
Q := \frac{-\pi}{\sqrt{2}} D^{2.5} \sqrt{\frac{g \cdot h_f}{L}} \cdot \log \left( \frac{\varepsilon}{3.7D} + 2.51v \cdot \sqrt{\frac{L}{2g \cdot h_f D^3}} \right)
\]

\[Q = 3.286 \times 10^{-3} \text{ m}^3/\text{s}\]

Note this barely changed!

Now calculate pressure at center of Tee

assume half of Tee losses to surge chamber

\[
h_{f1} := f' \cdot \frac{8}{\pi^2} \cdot \frac{L_1 \cdot Q^2}{d_1^5} \quad h_{f1} = 39.946 \text{ m}
\]

\[
h_{\text{minor}} := \frac{8 \cdot Q^2}{g \cdot \pi^2 \cdot d_1^4} \left( 0.5 \cdot K_{\text{tee}} \right) \quad h_{\text{minor}} = 0.071 \text{ m}
\]

total headloss to center of Tee is major plus minor

\[
h_{\text{loss1}} := h_{f1} + h_{\text{minor}} \quad h_{\text{loss1}} = 40.017 \text{ m}
\]

\[
P_{\text{pipe2}} := P_1 - h_{\text{loss1}} \cdot \rho \cdot g \quad P_{\text{pipe2}} = 7.563 \times 10^3 \text{ Pa}
\]
The algebra gets messy here. We need to find the final height of air, but that is a function of the pressure in the middle of the tee and the height of water above the tee. This dependency results in a quadratic equation. So below is my solution to the quadratic equation.

\[
\begin{align*}
a &:= \rho \cdot g \\
b &:= -h_t \cdot \rho \cdot g - P_{pipe2} - P_{atm} \\
c &:= h_t (P_{pipe2} + P_{atm}) - (P_1 + P_{atm} - \rho \cdot g \cdot h_{w1}) (h_t - h_{w1}) \\
h_{w2} &:= \frac{-b - \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \\
h_{w2} &= 0.041 \text{ m} \\
h_{a2} &:= h_t - h_{w2} \quad h_{a2} = 0.459 \text{ m} \\
\end{align*}
\]

Simplifying assumption... Neglect static pressure drop in water column, but don't neglect minor losses. Then pressure in air is pressure in pipe

\[
P_{\text{surge_absolute}} \cdot h_{a1} = P_{\text{surge_absolute}} \cdot h_{a2}
\]

\[
(P_{\text{atm}} + P_1) \cdot h_{a1} = (P_{\text{atm}} + P_{pipe2}) \cdot h_{a2} \quad P_1 = 4 \times 10^5 \text{ Pa} \\

h_{a2} := \frac{(P_{\text{atm}} + P_1) \cdot h_{a1}}{P_{\text{atm}} + P_{pipe2}} \\

h_{a2} = 0.46 \text{ m} \\

h_{w2} := h_t - h_{a2} \quad h_{w2} = 0.04 \text{ m}
\]