Prelim 2 2006

Make sure you clearly draw and label the control surfaces and the coordinate system.

**Problem 1**

*Find mass given h and Q*

\[
\begin{align*}
  h &:= 2 \text{ m} \\
  Q &:= 0.5 \text{ L/s} \\
  d_{\text{nozzle}} &:= 1 \text{ cm} \\
  \rho &:= 1000 \text{ kg/m}^3 \\
  \pi d_{\text{nozzle}}^2 / 4 &:= A_{\text{nozzle}} \\
  A_{\text{nozzle}} &:= 7.854 \times 10^{-5} \text{ m}^2 \\
  V_{\text{nozzle}} &:= \frac{Q}{A_{\text{nozzle}}} \\
  V_{\text{nozzle}} &:= 6.366 \text{ m/s} \\
\end{align*}
\]

Apply Bernoulli between the nozzle and the hemisphere

\[
\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2
\]

\[
V_{\text{nozzle}}^2 / 2g = V_{\text{hemisphere}}^2 / 2g + h
\]

\[
V_{\text{hemisphere}} := \sqrt{V_{\text{nozzle}}^2 - 2h \cdot g}
\]

\[
V_{\text{hemisphere}} := 1.141 \text{ m/s}
\]

Now apply the linear momentum equation to the hemisphere. Control surface one is where the jet hits the hemisphere. The other control surface is where the water leaves the hemisphere.

\[
M_{1z} := -\rho \cdot Q \cdot V_{\text{hemisphere}}
\]

\[
M_{2z} := -\rho \cdot Q \cdot V_{\text{hemisphere}} \quad \text{This is in the negative z direction}
\]

The pressure is atmospheric everywhere

\[
F_{ssz} := M_{1z} + M_{2z}
\]

\[
F_{ssz} := -2\rho \cdot Q \cdot V_{\text{hemisphere}}
\]
$F_{ssz} = -1.141 \text{ N}$  

This is the force of the hemisphere on the water

\[ M_{\text{hemisphere}} := \frac{-F_{ssz}}{g} \]

$M_{\text{hemisphere}} = 0.116 \text{ kg}$
What is the horizontal component of force that must be applied to the pipe bend to keep it from rotating?

$$Q := 1 \, \frac{m^3}{s}$$

$$d := 0.5m$$

$$p := 50k \cdot Pa$$

$$h := 4m$$

$$r := 1m$$

$$A := \frac{\pi d^2}{4} \quad A = 0.196 \, m^2$$

$$V := \frac{Q}{A} \quad V = 5.093 \, \frac{m}{s}$$

$$F_{p1x} := 0 \quad F_{p2x} := 0 \quad M_{1x} := 0$$

$$M_{2x} := -\rho \cdot Q \cdot V \quad M_{2x} = -5.1 \, kN$$

$$F_{ssx} := M_{1x} + M_{2x} - (F_{p1x} + F_{p2x})$$

$$F_{ssx} := M_{2x} \quad F_{ssx} = -5.1 \, kN$$

This is the force of the pipe bend on the water. This is the same as the force that must be applied to the pipe bend.

Below is an unnecessary calculation of a reasonable pressure at the pressure tap.

$$p_h := h \cdot \rho \cdot g \quad p_h = 39.227 \, k \cdot Pa$$

$$\frac{\rho V^2}{2} = 12.969 \, k \cdot Pa$$

$$p := p_h + \frac{\rho \cdot V^2}{2} \quad p = 52.196 \, k \cdot Pa$$