SURFACE DEFORMATION DUE TO SHEAR AND TENSILE FAULTS IN A HALF-SPACE

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ABSTRACT

A complete suite of closed analytical expressions is presented for the surface displacements, strains, and tilts due to inclined shear and tensile faults in a half-space for both point and finite rectangular sources. These expressions are particularly compact and free from field singular points which are inherent in the previously stated expressions of certain cases. The expressions derived here represent powerful tools not only for the analysis of static field changes associated with earthquake occurrence but also for the modeling of deformation fields arising from fluid-driven crack sources.

INTRODUCTION

Since dislocation theory was first introduced to the field of seismology by Stekete (1958), as well as a pioneer work by Rongved and Frasier (1968), numerous theoretical formulations describing the deformation of an isotropic homogeneous semi-infinite medium have been developed with increasing completeness and generality of source type and geometry. They range from the derivation of the surface displacement due to a point source of vertical strike-slip type in a Poisson solid (Stekete, 1958) to the strain fields at depth due to an inclined finite shear fault in a medium with arbitrary elastic constants (Iwasaki and Sato, 1979). The accomplishments of the various papers through which this progress has been achieved are summarized in Table 1.

Efforts to develop the formulations in a more realistic earth model have also been advanced through numerous studies, which include the effect of earth curvature (McGinley, 1969; Ben-Menahem et al., 1969; Smylie and Mainsina, 1971), the effect of surface topography (Ishii and Takagi, 1967a; Takemoto, 1981; Segall and McTague, 1984), the effect of crustal layering (Ishii and Takagi, 1967b; McGinley, 1969; Ben-Menahem and Gillon, 1970; Singh, 1970; Sato, 1971; Rybicki, 1971; Chinmenry and Jovanovich, 1972; Sato and Matsu'ura, 1973; Jovanovich et al., 1974a; b; Matsu'ura and Sato, 1975), the effect of lateral inhomogeneity (Rybicki, 1971, 1978; Rybicki and Kasahara, 1977; McHugh and Johnston, 1977; Niewiadomski and Rybicki, 1984), and the effect of obliquely layered medium (Sato, 1974; Sato and Yamashita, 1975). These studies revealed that the effect of earth curvature is negligible for the shallow events at distances of less than 20°, but that the vertical laying or lateral inhomogeneity can sometimes cause considerable effects on the deformation fields.

In spite of such an advance in calculating theoretical fields, the analyses of actual observations are still mostly based upon the simplest assumption of an isotropic homogeneous half-space and the simplest source configuration, largely for the following three reasons. First, it is most convenient and useful as the first approximation model. Second, the source model itself is inherently nonunique. Third, the quality of crustal movement data is generally poor at least up to the present.
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(Mikumo, 1973; Okada, 1980; Wyatt, 1982; Wyatt et al., 1984). The last two factors
often make it meaningless to compare the data with the predictions of an elaborate
source or earth model.

The first objective of this paper is to check and review the closed analytical
expressions which are already published to describe the surface deformation due to
shear fault in a half-space. As our observations are restricted to near-surface
fault, this class of solutions has the greatest practical importance to the study of the
earthquake sources. Some of them, as presented, are too lengthy and complicated,
while others have some singularities under the special conditions. For example,
Savage and Hastie's (1966) formula is too complicated and cannot be applied to the
vertical or horizontal fault, while Sato and Matsuur'a (1974) formula results in
"zero divide" at the points where the extensions of fault edges intersect the ground
surface. Besides, misprints occur all too often in the published expressions. In this
paper, the compact formulas to calculate the surface displacements, strains, and
 tilts due to a general shear fault in a half-space are given, which have been carefully
checked to be free from any singularities.

The second objective of this paper is to add a heretofore unknown solution for the
displacements, strains, and tilts arising from opening-mode dislocations. In
contrast to the progress that has been made in the modeling of the deformation
fields due to shear dislocations, the studies related to tensile fault are scarce as is
seen in Table 1. The main reason for this is, no doubt, the importance that has
been described to model the static field changes associated with earthquake
occurrence. Tensile fault representation, which has a Burger’s vector normal to the
dislocation surface, also has some very important geophysical applications, such as
a modeling of the deformation fields due to dyke injection in the volcanic region,
magmocrystalline, or fluid-driven cracks.

Berry and Sales (1962) derived the surface displacement fields due to a closure of
horizontal crack in a transversely isotropic medium. Maruyama (1964) gave the
expressions of surface displacements due to vertical and horizontal tensile faults in
a semi-infinite Poisson solid. Yamazaki (1978) treated the deformation fields arising
from a dilatancy source. Davis (1983) derived an expression of the vertical displace-
ment due to an inclined tensile fault in a half-space. He showed that this model can
approximate well a tensile crack, just as shear dislocations are successfully used to
approximate the deformation fields by shear cracks.

Recently, Evans and Wyatt (1984) found an interesting relation between changes in
the water-head within a borehole and associated ground surface deformation in
the surrounding region. Based upon the mechanism that subsurface hydraulically
conductive fractures respond to changing fluid pressure, they suggested a quanti-
tative tensile crack model to explain the observation. Their work has important
implications for the measurement of crustal deformation in that it provides a
physical basis for understanding an important class of crustal movement noise. It is
well known that the precipitation is a major source for crustal movement
observation not only in the short period but also in the long one (Kasahara et al.,
1983), and it is definite that the precipitation affects the ground movement through
some changes in the state of groundwater (Shichi and Okada, 1979; Edge et al.,
1983a, b; Takemoto, 1983). But so far, the effects of precipitation were mostly
discussed with appropriate formal mathematical models (Takemoto, 1967; Tanaka,
1967; Sato et al., 1986; Yanagisawa, 1980) or nonlinear tank model simulators.
\[ \left( \frac{\rho_1}{e_1} - \frac{\rho_2}{e_2} \right) \frac{1}{1 + \frac{\rho_1}{e_1}} + \left( \frac{\rho_2}{e_2} + \frac{\rho_2}{e_2} \right) \frac{1}{1 + \frac{\rho_2}{e_2}} \right] \Rightarrow \frac{d}{\eta} \]
\[ (1) \quad \text{For dip-slip} \]
\[ \text{Stress (2)} \quad \sigma + \sigma = \sigma \]

\[ \text{Displacements (1)} \quad \text{where} \]

\[ (2) \quad \text{For strike-slip} \]

\[ (3) \quad \text{For tectonic fault} \]

\[ (4) \quad \text{For topography} \]

\[ (5) \quad \text{For surface deformation in a half-space} \]

\[ \text{Note:} \quad \text{The above equations represent the stress and displacement components in a half-space model considering dip-slip, strike-slip, and tectonic fault scenarios.} \]
For tensile fault

\[
\begin{align*}
\frac{\partial u_0}{\partial x} &= U_0 \left[ \frac{3q^2}{2\pi R^5} \left(1 - \frac{5x^2}{R^2}\right) - J_0 \sin^2 \delta \right] \Delta \Sigma \\
\frac{\partial u_0}{\partial y} &= U_0 \left[ \frac{3xq}{2\pi R^5} \left(2\sin \delta - \frac{5yq}{R^2}\right) - J_0 \sin^2 \delta \right] \Delta \Sigma \\
\frac{\partial u_0}{\partial z} &= U_0 \left[ \frac{3xq}{2\pi R^5} ight] \left(q + 2y \sin \delta - \frac{5yq}{R^2}\right) \Delta \Sigma \\
\frac{\partial u_0}{\partial z} &= U_0 \left[ \frac{3q}{2\pi R^5} \right] \left(q + 2y \sin \delta - \frac{5yq}{R^2}\right) \Delta \Sigma \\
\end{align*}
\]

(15)

where \( s = p \sin \delta + q \cos \delta \) and

\[
\begin{align*}
J_0 &= \frac{\mu}{\lambda + \mu} \left[ -3xy \frac{3R + d}{R(R + d)^3} + 3x^2y \frac{5R^2 + 4Rd + d^2}{R^3(R + d)^3} \right] \\
J_1 &= \frac{\mu}{\lambda + \mu} \left[ -3xy \frac{R}{R + d} + 3x^2 \frac{5R^2 + 4Rd + d^2}{R^3} \right] \\
J_2 &= \frac{\mu}{\lambda + \mu} \left[ -3xy \frac{R}{R^2} \right] \\
J_3 &= \frac{\mu}{\lambda + \mu} \left[ -3xy \frac{R}{R^2} \right] - J_1 \\
J_4 &= \frac{\mu}{\lambda + \mu} \left[ -3xy \frac{R}{R^2} \right] - J_0 \\
\end{align*}
\]

(16)

(3) Tilts

For strike-slip

\[
\begin{align*}
\frac{\partial u_0}{\partial x} &= -U_0 \left[ \frac{3xy}{R^5} \right] \left(1 - \frac{5x^2}{R^2}\right) + K_0 \sin \delta \Delta \Sigma \\
\frac{\partial u_0}{\partial y} &= -U_0 \left[ \frac{15xy}{R^5} \right] + \left(\frac{3xy}{R^5} + K_0\right) \sin \delta \Delta \Sigma.
\end{align*}
\]

(17)

For dip-slip

\[
\begin{align*}
\frac{\partial u_0}{\partial x} &= -U_0 \left[ \frac{15xy}{R^5} \right] - \left(\frac{5xy}{R^5} - K_0\sin \delta \cos \delta \right) \Delta \Sigma \\
\frac{\partial u_0}{\partial y} &= -U_0 \left[ \frac{5y}{R^5} \right] - \left(\frac{5y}{R^5} - K_0\sin \delta \cos \delta \right) \Delta \Sigma.
\end{align*}
\]

(18)

\[
\frac{\partial u_0}{\partial x} = \frac{U_0}{2\pi R^5} \left(-\frac{15xy}{R^2} - K_0 \sin \delta \frac{\sin \delta}{R^2}\right) \Delta \Sigma
\]

(19)

where

\[
\begin{align*}
K_0 &= -\frac{\mu}{\lambda + \mu} \left[ \frac{2R + d}{R^2(R + d)^2} - \frac{y^2 8R^2 + 9Rd + 3d^2}{R^3(R + d)^3} \right] \\
K_1 &= -\frac{\mu}{\lambda + \mu} \left[ \frac{2R + d}{R^2(R + d)^2} - \frac{y^2 8R^2 + 9Rd + 3d^2}{R^3(R + d)^3} \right] \\
K_2 &= -\frac{\mu}{\lambda + \mu} \frac{3xd}{R^5} - K_1 \\
K_3 &= -\frac{\mu}{\lambda + \mu} \frac{3xd}{R^5} - K_2 \\
\end{align*}
\]

(20)

FINITE RECTANGULAR SOURCE

For a finite rectangular fault with length \( L \) and width \( W \) (Figure 1), the deformation field can be derived by taking \( x - \xi' \), \( y - \eta' \cos \delta \) and \( d - \eta' \sin \delta \) in place of \( x, y \), and \( d \) in the equations obtained in the previous section and by performing the integration

\[
\int_0^L \int_0^L \Delta \Sigma' \int_0^W \Delta \Sigma' \int_0^\infty d\eta'.
\]

(21)

Following Sato and Matsu'ura (1974), it is convenient to change variables from \( \xi', \eta' \) to \( \xi, \eta \) by

\[
\begin{align*}
x - \xi' &= \xi \\
p - \eta' &= \eta
\end{align*}
\]

(22)

where, \( p = y \cos \delta + d \sin \delta \) as before. Then, the above integration becomes

\[
\int_0^L \int_0^W \int_0^\infty d\xi' \int_0^\infty d\eta'.
\]

(23)

The final results condensed into compact forms are listed below using Chinnery's notation \( \| \) to represent the substitution

\[
f(\xi, \eta) \| = f(x, p) - f(x, p - W) - f(x - L, p) + f(x - L, p - W).
\]

(24)

If we take a rectangular fault with length \( 2L \) (dashed line in Figure 1), it is only necessary to replace \( x \) in the first and the second terms of the right-hand side of equation (24) to \( x + L \).
(1) Displacements

For strike-slip

\[ \begin{align*}
    u_x &= -\frac{U_1}{2\pi} \left[ \frac{\xi q}{R(R + \eta)} + \tan^{-1} \frac{\xi q}{R^2} + I_1 \sin \delta \right] \\
    u_y &= -\frac{U_1}{2\pi} \left[ \frac{\eta q}{R(R + \eta)} + q \cos \delta + I_2 \sin \delta \right] \\
    u_z &= -\frac{U_1}{2\pi} \left[ \frac{\delta q}{R(R + \eta)} + q \sin \delta + I_3 \sin \delta \right].
\end{align*} \]

(25)

For dip-slip

\[ \begin{align*}
    u_x &= -\frac{U_2}{2\pi} \left[ \frac{q}{R} - I_4 \sin \delta \cos \delta \right] \\
    u_y &= -\frac{U_2}{2\pi} \left[ \frac{\eta q}{R(R + \xi)} + \cos \delta \tan^{-1} \frac{\xi q}{R^2} - I_1 \sin \delta \cos \delta \right] \\
    u_z &= -\frac{U_2}{2\pi} \left[ \frac{\delta q}{R(R + \xi)} + \sin \delta \tan^{-1} \frac{\eta q}{R^2} - I_3 \sin \delta \cos \delta \right].
\end{align*} \]

(26)

For tensile fault

\[ \begin{align*}
    u_x &= -\frac{U_3}{2\pi} \left[ \frac{q^2}{R(R + \eta)} - I_5 \sin^2 \delta \right] \\
    u_y &= -\frac{U_3}{2\pi} \left[ -\frac{\delta q}{R(R + \xi)} - \sin \delta \left( \frac{\xi q}{R(R + \eta)} - \tan^{-1} \frac{\xi q}{R^2} \right) - I_4 \sin^2 \delta \right] \\
    u_z &= -\frac{U_3}{2\pi} \left[ \frac{\eta q}{R(R + \xi)} + \cos \delta \left( \frac{\xi q}{R(R + \eta)} - \tan^{-1} \frac{\eta q}{R^2} \right) - I_3 \sin^2 \delta \right]
\end{align*} \]

(27)

where

\[ \begin{align*}
    I_1 &= \frac{\mu}{\lambda + \mu \cos \delta} \left[ \frac{-1}{R + d} \right] - \sin \delta \cos \delta I_5 \\
    I_2 &= \frac{\mu}{\lambda + \mu} \left[ -\ln(R + \eta) \right] - I_3 \\
    I_3 &= \frac{\mu}{\lambda + \mu \cos \delta} \left[ \frac{1}{R + d} - \ln(R + \eta) \right] + \sin \delta \cos \delta I_4 \\
    I_4 &= \frac{\mu}{\lambda + \mu \cos \delta} \left[ \ln(R + d) - \sin \delta \ln(R + \eta) \right] \\
    I_5 &= \frac{\mu}{\lambda + \mu \cos \delta} \left[ \frac{q^2}{R(R + \eta)} - \sin \delta \cos \delta \right].
\end{align*} \]

(28)

and if \( \cos \delta = 0 \),

\[ \begin{align*}
    I_1 &= -\frac{\mu}{2(\lambda + \mu)} \left[ \frac{\xi q}{R} \right] \\
    I_3 &= -\frac{\mu}{2(\lambda + \mu)} \left[ \frac{\eta q}{R} \right] \\
    I_4 &= -\frac{\mu}{\lambda + \mu} \left[ \frac{q}{R + d} \right] \\
    I_5 &= -\frac{\mu}{\lambda + \mu} \left[ \frac{\xi q}{R + d} \right]
\end{align*} \]

(29)

\[ \begin{align*}
    p &= \gamma \cos \delta + d \sin \delta \\
    q &= \gamma \sin \delta - d \cos \delta \\
    \tilde{\gamma} &= \eta \cos \delta + q \sin \delta \\
    \tilde{d} &= \eta \sin \delta - q \cos \delta \\
    R^2 &= \xi^2 + \eta^2 + q^2 = \xi^2 + \tilde{\gamma}^2 + \tilde{d}^2 \\
    X^2 &= \xi^2 + q^2.
\end{align*} \]

(30)

When \( \cos \delta = 0 \), we must be careful that there are two cases of \( \sin \delta = +1 \) and \(-1\).

(2) Strains

For strike-slip

\[ \begin{align*}
    \frac{\partial u_x}{\partial x} &= -\frac{U_1}{2\pi} \left[ \xi^2 q A_x - J_1 \sin \delta \right] \\
    \frac{\partial u_y}{\partial y} &= -\frac{U_1}{2\pi} \left[ \frac{\xi q}{R^2} \right] - \left( \xi^2 A_x + J_2 \right) \sin \delta \\
    \frac{\partial u_z}{\partial x} &= -\frac{U_1}{2\pi} \left[ \frac{\xi q}{R^2} \cos \delta + (\xi^2 A_x - J_2) \sin \delta \right] \\
    \frac{\partial u_z}{\partial y} &= -\frac{U_1}{2\pi} \left[ \frac{\xi q}{R^2} \cos \delta + \ight] \\
    \frac{\partial u_z}{\partial y} &= -\frac{U_1}{2\pi} \left[ \frac{\xi q}{R^2} \cos \delta + \right] \\
    \frac{\partial u_z}{\partial y} &= -\frac{U_1}{2\pi} \left[ \frac{\xi q}{R^2} \cos \delta + \right] \\
    \frac{\partial u_z}{\partial y} &= -\frac{U_1}{2\pi} \left[ \frac{\xi q}{R^2} \cos \delta + \right]
\end{align*} \]

(31)
For dip-slip
\[
\begin{align*}
\frac{\partial u_{x}^0}{\partial x} &= U_3 \left[ \frac{\xi q}{R^3} + J_3 \sin \delta \cos \delta \right] \\
\frac{\partial u_{y}^0}{\partial y} &= U_3 \left[ \frac{\dot{\gamma} q}{R^3} - \frac{\sin \delta}{R} + J_3 \sin \delta \cos \delta \right] \\
\frac{\partial u_{x}^0}{\partial x} &= U_3 \left[ \frac{\dot{\gamma} q}{R^3} - \frac{q \cos \delta}{R(R + \eta)} + J_3 \sin \delta \cos \delta \right] \\
\frac{\partial u_{y}^0}{\partial y} &= U_3 \left[ \frac{\dot{\gamma} q}{R^3} - \frac{2\dot{\gamma} q}{R(R + \xi)} + \frac{\xi \cos \delta}{R(R + \eta)} \right] \sin \delta + J_3 \sin \delta \cos \delta. \tag{32}
\end{align*}
\]

For tensile fault
\[
\begin{align*}
\frac{\partial u_{x}}{\partial x} &= -\frac{U_3}{2\pi} \left[ \xi q^2 A_3 + J_3 \sin^2 \delta \right] \\
\frac{\partial u_{y}}{\partial y} &= -\frac{U_3}{2\pi} \left[ \frac{\dot{\gamma} q}{R^3} - \frac{\xi q A_3}{R} + J_3 \sin^2 \delta \right] \\
\frac{\partial u_{x}}{\partial x} &= -\frac{U_3}{2\pi} \left[ \frac{q^2}{R^3} \cos \delta + \frac{\xi q A_3}{R} + J_3 \sin^2 \delta \right] \\
\frac{\partial u_{y}}{\partial y} &= -\frac{U_3}{2\pi} \left[ \frac{\dot{\gamma} q}{R^3} - \frac{\xi q A_3}{R} - \frac{q \sin^2 \delta}{R(R + \xi)} - (\xi q A_3 - J_2) \sin^2 \delta \right]. \tag{33}
\end{align*}
\]

where
\[
\begin{align*}
J_1 &= \frac{\mu}{\lambda + \mu} \frac{\sin \delta}{\cos \delta} \left[ \frac{\xi^2}{R(R + d)^2} + \frac{1}{R + d} \right] - K_3 \\
J_2 &= \frac{\mu}{\lambda + \mu} \frac{\sin \delta}{\cos \delta} \left[ \frac{\xi \dot{\gamma}}{R(R + d)^2} \right] - K_1 \\
J_3 &= \frac{\mu}{\lambda + \mu} \left( \frac{\xi}{R(R + \eta)} \right) - J_2 \\
J_4 &= \frac{\mu}{\lambda + \mu} \left( \frac{\cos \delta}{R} - \frac{\xi q A_3}{R(R + \eta)} \right) - J_1. \tag{34}
\end{align*}
\]

\[
J_1 = \frac{\mu}{2(\lambda + \mu)} \frac{\xi^2}{R(R + d)^2} - \frac{1}{R + d} \sin \delta - \frac{1}{R(R + d)^2} \cos \delta K_3
\]

\[
J_2 = \frac{\mu}{\lambda + \mu} \frac{\sin \delta}{\cos \delta} \left[ \frac{\xi \dot{\gamma}}{R(R + d)^2} \right] - K_1
\]

\[
J_3 = \frac{\mu}{\lambda + \mu} \left( \frac{\xi}{R(R + \eta)} \right) - J_2
\]

\[
J_4 = \frac{\mu}{\lambda + \mu} \left( \frac{\cos \delta}{R} - \frac{\xi q A_3}{R(R + \eta)} \right) - J_1.
\tag{34}
\]

\[
J_1 = \frac{\mu}{2(\lambda + \mu)} \frac{\xi^2}{R(R + d)^2} - \frac{1}{R + d} \sin \delta - \frac{1}{R(R + d)^2} \cos \delta K_3
\]

\[
J_2 = \frac{\mu}{\lambda + \mu} \frac{\sin \delta}{\cos \delta} \left[ \frac{\xi \dot{\gamma}}{R(R + d)^2} \right] - K_1
\]

\[
J_3 = \frac{\mu}{\lambda + \mu} \left( \frac{\xi}{R(R + \eta)} \right) - J_2
\]

\[
J_4 = \frac{\mu}{\lambda + \mu} \left( \frac{\cos \delta}{R} - \frac{\xi q A_3}{R(R + \eta)} \right) - J_1.
\tag{34}
\]

\[
\begin{align*}
J_1 &= \frac{\mu}{2(\lambda + \mu)} \left( \frac{\xi^2}{R(R + d)^2} - \frac{1}{R + d} \sin \delta - \frac{1}{R(R + d)^2} \cos \delta K_3 \right) \\
J_2 &= \frac{\mu}{\lambda + \mu} \left( \frac{\sin \delta}{\cos \delta} \left[ \frac{\xi \dot{\gamma}}{R(R + d)^2} \right] - K_1 \right) \\
J_3 &= \frac{\mu}{\lambda + \mu} \left( \frac{\xi}{R(R + \eta)} \right) - J_2 \\
J_4 &= \frac{\mu}{\lambda + \mu} \left( \frac{\cos \delta}{R} - \frac{\xi q A_3}{R(R + \eta)} \right) - J_1.
\end{align*}
\]

(3) Tilts

For strike-slip
\[
\begin{align*}
\frac{\partial u_{x}}{\partial x} &= U_3 \left[ -\xi q^2 A_3 \cos \delta + \left( \frac{\xi q}{R^3} - K_3 \right) \sin \delta \right] \\
\frac{\partial u_{y}}{\partial y} &= U_3 \left[ \frac{\dot{\gamma} q}{R^3} \cos \delta + \left( \xi q A_3 \cos \delta - \frac{\sin \delta}{R} + \frac{\xi \dot{\gamma} q}{R^3} \right) \sin \delta \right]. \tag{37}
\end{align*}
\]

For dip-slip
\[
\begin{align*}
\frac{\partial u_{x}}{\partial x} &= U_3 \left[ \frac{\xi q}{R^3} + \frac{q \sin \delta}{R(R + \eta) + K_3 \sin \delta \cos \delta} \right] \\
\frac{\partial u_{y}}{\partial y} &= U_3 \left[ \frac{\dot{\gamma} q}{R^3} + \frac{\xi \sin \delta}{R(R + \eta)} \right] - \frac{2q}{R(R + \xi)} \sin \delta + K_3 \sin \delta \cos \delta. \tag{38}
\end{align*}
\]

For tensile fault
\[
\begin{align*}
\frac{\partial u_{x}}{\partial x} &= -U_3 \left[ \frac{\dot{\gamma} q}{R^3} - \xi q A_3 \cos \delta + K_3 \sin \delta \cos \delta \right] \\
\frac{\partial u_{y}}{\partial y} &= -U_3 \left[ \frac{\dot{\gamma} q}{R^3} + \xi q A_3 \cos \delta - \frac{q \sin \delta}{R(R + \eta)} \right] - \frac{2q}{R(R + \xi)} - K_3 \sin \delta \cos \delta. \tag{39}
\end{align*}
\]
where

\[
\begin{align*}
K_1 &= \frac{\mu}{\lambda + \mu} \frac{\xi}{R(R + \eta)} \left[ \frac{1}{R(R + \eta) + \delta} - \sin \delta \right] \\
K_2 &= \frac{\mu}{\lambda + \mu} \left[ -\frac{\sin \delta}{R} + \frac{\xi}{R(R + \eta)} \right] - K_3 \\
K_3 &= \frac{\mu}{\lambda + \mu} \frac{1}{R(R + \eta) + \delta} \left[ \frac{\xi}{R(R + \eta)} - \frac{\delta}{R(R + \eta) + \delta} \right]
\end{align*}
\]

(40)

and if \( \cos \delta = 0 \),

\[
\begin{align*}
K_1 &= \frac{\mu}{\lambda + \mu} \frac{\xi q}{R(R + \eta) + \delta} \\
K_2 &= \frac{\mu}{\lambda + \mu} \frac{\sin \delta}{R(R + \eta) + \delta} \left[ \frac{\xi^2}{R(R + \eta) + \delta} - 1 \right].
\end{align*}
\]

(41)

In the indefinite integral expressions stated in this section, some terms become singular at the special conditions. Returning to the integral (23) and carefully checking these special cases, we can reach the following rules to avoid all the singularities. (i) When \( q = 0 \), set \( \tan^{-1}(\xi q/R) \) = 0 in equations (25) to (27). (ii) When \( \xi = 0 \), set \( K_3 = 0 \) in equation (28). (iii) When \( R + \eta = 0 \) (this occurs only when \( \sin \delta < 0 \) and \( \xi = q = 0 \)), set all the terms which contain \( R + \eta \) in their denominators to be zero in equations (25) to (40), and replace \( \ln(R + \eta) \) to \( -\ln(R - \eta) \) in equations (28) and (29).

**DISCUSSION**

A compact analytical expression of the surface displacements, strains, and tilts due to inclined shear and tensile faults in a half-space are given for both point and finite rectangular sources in the preceding sections. All similar expressions known to the author were checked to be equivalent to the formulas given here except for some misprints in the literatures, which are now listed in the Appendix.

The formulas for point sources derived here can be used as an alternative of Maruyama's (1964) expressions to estimate far-field deformation or to construct the deformation fields by more general faults. The formulas for finite shear fault derived here are essentially identical to those of Matsuura (1977) as to the displacements and Sato and Matsuura (1974) as to the strains and tilts. But here, some revisions have been made to overcome the following difficulties which are included in the previous expressions. (i) On the line where the extension of the fault plane intersects the ground surface, the displacement becomes singular. (ii) On the lines where the vertical planes containing the inclined edges of the fault intersect the ground surface, the vertical displacement becomes singular. (iii) Displacements cannot be evaluated in case of \( \delta = -\pi/2 \). (iv) At the points where the inclined edges of the fault intersect the ground surface, the strains and tilts become singular. In addition to this revision, the formulas for tensile fault are newly added in this paper, and the work to derive the expressions of the surface deformation fields due to buried rectangular faults in a half-space seems to have come to maturity now.
As to the surface deformation due to more general polygon-shaped faults, we can use the results by Comminou and Dundurs (1975). They gave the expressions of displacement and strain at the free surface of a half-space for an angular dislocation. Any polygon-shaped faults (shear or tensile) can be constructed by a superposition of a finite number of angular dislocations.

All the formulas obtained here are composed of the terms of two kinds; ones independent of the medium constants, $\lambda$ and $\mu$ and the others dependent on them. The latter which are denoted by $I$, $J$, or $K$ appear in the same fashion in the formulas of dip-slip case and tensile-fault case. This can be realized by an analogy with the $P-SV$ coupling in the seismic wave theory, whereas the $SH$ wave corresponds to the strike-slip case. It is clear that the deformation fields produced by a vertical fault of dip-slip type and the ones produced by a horizontal fault of any type do not depend on the medium constants, $\lambda$ and $\mu$.

The $z$ direction strain components were not given in the preceding sections, but they can be easily found as follows using the boundary conditions at the free surface.

\[
\frac{\partial u_x}{\partial z} = -\frac{\partial U_z}{\partial x},
\]

\[
\frac{\partial u_y}{\partial z} = -\frac{\partial U_z}{\partial y},
\]

\[
\frac{\partial u_z}{\partial z} = -\frac{\lambda}{\lambda + 2\mu} \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right).
\]

To assist the development of a computer program based upon these expressions, several numerical results to check it are listed in Table 2. Here, case 1 is for the point source, and the others are for the finite rectangular sources. A medium is assumed to be $\lambda = \mu$ in all the cases, and the results are presented in the unit of $U \Delta S$ in case 1 and in the unit of $U$ in the others, where $U$ stands for $U_1$, $U_2$, or $U_3$.

ACKNOWLEDGMENTS

This work was accomplished during the author's stay at the Lamont-Doherty Geological Observatory of Columbia University. The author is grateful to Dr. Keith F. Evans of the LDGO for critical reading of the manuscript and for many suggestive discussions. He would also like to thank Dr. R. John Bean of the LDGO for valuable discussion and encouragement in the course of this study. The author appreciates several comments by the reviewer, as well as the ones by Professor T. Mikumo of Disaster Prevention Research Institute, Kyoto University, Dr. J. C. Savage, of the U.S. Geological Survey, Dr. H. Ishii of Earthquake Research Institute, University of Tokyo, and Dr. K. Yamazaki of Tokyo Ongaku University. They were very helpful in making this paper clear.

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Manuscript received 16 November 1984
REGIONAL ARRAYS AND OPTIMUM DATA PROCESSING SCHEMES

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ABSTRACT

In the 1960's, Lincoln Laboratory, Texas Instruments, and Teledyne-Geotech research groups were assigned tasks related to the design of the large-aperture arrays LASA and NORSAR resulting in a number of sophisticated processing techniques for additional signal-to-noise ratio (SNR) enhancements relative to conventional delay-and-sum processing (or beamforming). At that time, these techniques (essentially variations of Wiener filtering theory) did not prove a great success, and part of the blame was attributed to inadequate computing power for handling large data volumes in a nontrivial manner. However, with the present advent of miniarrays and relatively much faster computers, optimum array processing techniques are again in vogue. In this respect, we have examined different weighting schemes based on the noise structure as manifested in the noise covariance matrix using data from the prototype NORESS array in Norway. Results are as follows: (i) with strong correlations between sensors the processing gain is obtained essentially by deleting “redundant” sensors; (ii) with strong-tomoderate negative correlations, SNR gain in excess of \( \sqrt{N} \) is obtained by giving large weights to a few sensors to ensure destructive interference; (iii) with weakly structured noise conventional beamforming is very efficient as expected; and (iv) observed noise correlation curves exhibit rather strong spatial variation, i.e., depend on both phase velocity and azimuth. Also the time stationarity of the noise covariance matrix is rather weak, thus diminishing the expected gain from the optimum weighting schemes in a real-time context. Some experiments were also performed with a simplified maximum-likelihood filtering processor, and in this case approximate \( \sqrt{N} \) gains were obtained even for strongly correlated noise. Because processing gains implicitly reflect sensor geometry, criteria for array configurations are discussed in the light of the results obtained from the optimum processing experiments. Essentially, an array aperture of 3 km, like that of NORESS, represents high-pass filters with a lower cut-off at about 2 to 3 Hz while signal decorrelation and signal beamforming losses ensure that the array acts as a low-pass filter with a cut-off at about 6 Hz. However, using subsets of the array's original 25 sensors with correspondingly smaller apertures, the array's operational bandwidth can efficiently be shifted toward higher frequencies, say 4 to 8 Hz. Besides array aperture, the sensor interspacing distribution is an important parameter for judging array performance. SNR spectra for events with signal paths in oceanic and active tectonic belt regions peak at about 3 Hz, and for shield areas between 5 to 7 Hz. Finally, likely future array developments are discussed in view of recent initiatives within the seismological community concomitant with advances in communication and microprocessor technologies.

INTRODUCTION

The development of elaborate arrays of seismometers was initially politically motivated by the need for improved capability to study weak teleseismic events for nuclear test monitoring and associated seismological research. These efforts, formally dating back to a meeting of experts in Geneva in 1958 as part of the United Nations disarmament negotiations, culminated with the development of two large-aperture arrays; LASA in Montana (1965) and NORSAR in southeast Norway

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