IR-MAD
Iteratively Re-weighted Multivariate Alteration Detection


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Iteratively Re-weighted Multivariate Alteration Detection


- Effective at identifying areas of no-change, even when the fraction of invariant pixels is relatively small.
- Effective at identifying areas of significant change (radiometrically)
- No requirement for atmospheric correction (normalization)
- Code freely available

IR-MAD: Iteratively Re-weighted Multivariate Alteration Detection

- Two N-band multispectral images of the same scene acquired at different times. Ground reflectance changes have occurred at some locations, but not everywhere.

- Represent observations in the first N-band image by a random vector,
  \[ F = (F_1, \ldots, F_N)^T \]
  and create a scalar image, characterized by the random variable:
  \[ U = a^T F \]

- Then do the same for the second image:
  \[ G = (G_1, \ldots, G_N)^T \]
  \[ V = b^T G \]

- Now consider the scalar difference image: \( U - V \)
  *This combines all of the change information into one image.*

\[ \Rightarrow \text{Need for } a \text{ and } b \text{ to be specified in a suitable way.} \]

Nielsen et al. (1998) suggested Canonical Correlation Analysis (CCA)
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**Image 1:** \(F = (F_1, \ldots, F_N)^T\)  \(\Rightarrow\) linear combination: \(U = a^T F\)

**Image 2:** \(G = (G_1, \ldots, G_N)^T\)  \(\Rightarrow\) linear combination: \(V = b^T G\)

This leads to the coupled eigenvalue problem:

\[
\sum_{fg} \sum_{gg}^{-1} \sum_{gf} a = \rho^2 \sum_{ff} a \quad (1)
\]

\[
\sum_{gf} \sum_{ff}^{-1} \sum_{fg} b = \rho^2 \sum_{gg} b
\]

Where

- \(\Sigma_{ff}\) and \(\Sigma_{gg}\) are the covariance matrices of the two images
- \(\Sigma_{fg} = \Sigma_{gf}^T\) is the inter-image covariance matrix
- \(\rho = \text{vector of canonical correlations}\) where \(\rho_i = \text{corr}(U_i, V_i)\), are the square roots of the eigenvalues of the coupled eigenvalue problem.

The CVs are ordered by similarity (correlation).
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Image 1: \( F = (F_1 \ldots, F_N)^T \) \( \Rightarrow \) linear combination: \( U = a^T F \)

Image 2: \( G = (G_1 \ldots, G_N)^T \) \( \Rightarrow \) linear combination: \( V = b^T G \)

Solution of the coupled eigenvalue problems (Eq. 1) generates new multispectral images:

\[
\begin{align*}
U &= (U_1, \ldots, U_N)^T \\
V &= (V_1, \ldots, V_N)^T
\end{align*}
\]

the components of which are called the canonical variates (CVs), and are ordered by similarity (correlation).

The canonical correlations \( \rho_i = corr(U_i, V_i) \), \( i=1,\ldots, N \), are the square roots of the eigenvalues of the coupled eigenvalue problem

and the coefficients \( a_i \) and \( b_i \), \( i=1,\ldots, N \), are the eigenvectors which determine \( U \) and \( V \) from \( F \) and \( G \).

**The pair \( (U_1, V_1) \) is maximally correlated.**

The pair \( (U_2, V_2) \) is maximally correlated subject to being orthogonal to (and uncorrelated with) both \( U_1 \) and \( V_1 \)
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The MAD variates are generated as a sequence of transformed difference images:

\[ M_i = U_{N-i+1} - V_{N-i+1}, \quad i = 1, \ldots, N \]

and have statistical properties which make them very useful for visualizing and analyzing change information. They:

1. are uncorrelated: \( \text{cov}(M_i, M_j) = 0 \) for \( i \neq j \)
2. have successively decreasing variances:
   \[ \text{var}(M_i) = \sigma^2_{M_i} = 2(1 - \rho_{N-i+1}) \]
3. are ideally normally distributed (central limit theorem)
4. are invariant under affine transformation (Canty et al., 2004)
**IR-MAD**: Iteratively Re-weighted Multivariate Alteration Detection

**MAD variates**: \[ M_i = U_{N-i+1} - V_{N-i+1} , \quad i = 1, \ldots, N \]

**The first MAD variate**: \[ M_1 = U_N - V_N \]

has the greatest variance and, ideally, describes the maximum change information.

**The second MAD variate**: \[ M_2 = U_{N-1} - V_{N-1} \]

Has maximum variance subject to the condition that it is statistically uncorrelated with \( M_1 \)

**The \( \chi^2 \) image** represents the probability of change for each pixel. Each observation is weighted by a no-change probability given by

\[ \Pr(\text{no change}) = 1 - P_{\chi^2,p}(z) \]

\( \Pr \) (no change) is the probability that a sample \( z \) drawn from the chi-square distribution could be that large or larger. A small \( z \) implies a large probability of no change.
IR-MAD: *Iteratively* Re-weighted Multivariate Alteration Detection

In the presence of genuine change, it should be possible to improve the sensitivity of the MAD transformation by placing emphasis on establishing an increasingly better background of no change against which to detect change. This can be done using an iteration scheme in which observations are weighted by the probability of no change, as determined by the preceding iteration, when estimating the sample means and covariance matrices which determine – via CCA – the MAD variates for the next iteration.

**Iteration:**

- After one transformation, the first principal axis only corresponds *approximately* to the (highly correlated) no-change pixels, since it is determined by both no-change and change observations.

- Weighting the observations inversely to their distance from the first principal axis and repeating the transformation improves the position of the principal axes relative to the no-change pixels and, correspondingly, enhances the change signal.
IR-MAD: Iteratively Re-weighted Multivariate Alteration Detection

July 30, 2001

May 22, 2005

ASTER images near Esfahan, Iran
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ASTER images near Esfahan, Iran

Mar 11, 2002
Apr 9, 2001
May 22, 2005
July 30, 2001
Sep 11, 2005
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ASTER images near Esfahan, Iran

Mar 11, 2002
Apr 9, 2001
May 22, 2005
July 30, 2001
Sep 11, 2005
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June 26, 2001

August 29, 2001

Landsat ETM+ images over Jülich, Germany
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RGB color composite of MAD variates 4 (blue), 5 (green) and 6 (red) applied to the June 26, 2001 and August 29, 2001 Jülich, Germany images.