Lab #3 Conservation Equations and the Hydraulic Jump
CEE 331 Fall 2003

Safety
The major safety hazard in this laboratory is a shock hazard. Given that you will be working with water and items running on standard line voltages (the computer) you should pay attention to the possibility of electric shock. The two flumes have a few very small leaks and there will be a few wet spots under and around the flumes. If water gets near a 110-Volt electrical connection DO NOT clean it up. Seek a TA, Cowen, or one of the CEE technicians (Lee Virtue, Cameron Willkens who have offices across from the lab) for help.

The flow rates in this lab are sufficient to generate moderate forces. Take care with the pressure transducer, Pitot tubes, rulers, and any other items that you bring in contact with the flowing water as they may receive sufficient force to knock them around to unexpected locations.

Always work with a minimum of two people.

Objectives
In this laboratory you will investigate an open-channel flow (flow down a channel with a free-surface, e.g., not confined by a rigid surface as would be the case in pipe flow) using conservation equations (mass, linear momentum and energy). You will be introduced to the hydraulics phenomenon known as the hydraulic jump (see Figure 1) – the sudden transition from a higher energy state to a lower energy state while conserving momentum (analogous to a shock wave in compressible gas flows). This is your chance to get a tangible sense of these conservation equations and concepts such as the energy grade line and hydraulic grade line. You will also get a chance to think about the energy equation and when the assumptions of the Bernoulli equation are valid and when they are violated.

Theory

The Flow
We will use a sluice gate to convert potential energy to kinetic energy and create what is known as supercritical flow. We will study supercritical flow in some detail in the final weeks of the semester. The concept of supercritical flow is fairly straightforward (and covered in your text in Chapter 10 sections 10.1 and 10.2), and simply defined means that waves can only travel downstream. As an example consider throwing a rock into a slowly moving river. A circular wave pattern forms initially and propagates radially outward (until the banks are hit at least). If the river flow is slow then a significant portion of the circular wave pattern will propagate upstream – e.g., the waves make progress back against the river current with respect to the riverbank. This state is known as the subcritical state. Now, if the river speed increases eventually it will flow fast enough that none of the circular wave pattern will make progress upstream. When this scenario is true the river flow is said to be supercritical. For waves that have a long wave length with respect to their flow depth we can find the wave speed simply as

\[ c = \sqrt{gh} \]
where $h$ is the local flow depth and $g$ is the acceleration of gravity. Hence we can express the condition that the wave speed is equal to the flow speed as $V = c$ or

$$Fr = \frac{V}{\sqrt{gh}} = 1$$

where Fr is known as the Froude number and $V$ is the local flow velocity.

As suggested above a flow can exist in either a supercritical or subcritical state. For a flow, such as in our case, that is locally supercritical, downstream conditions may require the flow to increase its depth, say to get over a sill or weir. There is only one valid supercritical and one valid subcritical depth of the flow so if it must adjust its depth it must transition to the subcritical state, which is slower and hence deeper (to pass the same flow rate). Now normally over short reaches of a river it is reasonable to ignore energy loss and assume energy is constant. However, there is no way for a flow to decelerate smoothly and adjust to a subcritical state (note that it can accelerate smoothly from sub- to supercritical as it did beneath the sluice gate). It accomplishes this transition to deeper flow depth by a feature known as a hydraulic jump (see section 10.6 of your text), which as you will see is a highly turbulent event and dissipates significant energy.

In this lab we have set up a weir (an overflow gate as opposed to a sluice gate which is an underflow gate) at the outlet of our two teaching flumes in the environmental fluid mechanics teaching laboratory. This weir requires that the flow be at minimum 8 cm deep to get out of the flume and return to the pumps that supply the flume with water. Hence if we use a sluice gate to deliver a supercritical flow of depth about 1 – 2.5 cm to the flume, it must transition to a deeper state to get over the weir and out of the flume and it does this via a hydraulic jump. Figure 1 below shows an example of the flow.

**Constant Head Tank**

The flow is established via a constant head tank. This is a simple source condition that enforces a constant elevation head of water in a reservoir be available to drive a flow. The facility we are using (pictured in Figure 1) creates a constant head tank by filling the head box (left hand metal tank attached to flume looking at Figure 1) with water from a pump at a rate greater than the water flows out under the sluice gate (the sluice gate is the right-hand face of the head-box looking at Figure 1). Excess flow is drained out of the tank by a drain set at approximately 0.75 m above the facility’s test section bottom elevation. This water is returned to the inlet tank for the pump (PVC pipe seen in figure 1 just behind the head box that drains into the white plastic cylindrical tank). Hence you have a roughly constant head as long as this PVC pipe is draining some water. For the moment use this flume only and we will have the second (yellow flume) running shortly.

**Sluice Gate**

We can analyze the sluice gate flow assuming the flow is inviscid and therefore energy is conserved. Hence we can invoke Bernoulli’s equation and conservation of mass to find the velocity and flow rate that is delivered just downstream of the sluice gate. As discussed in class and in section 3.6.3 of the text we do not expect the flow to make the right angle turn under the sluice gate and hence the actual flow rate delivered is less than the sluice gate opening. The ratio
of the actual flow depth (say $h$) to the sluice gate opening (say $d$) is found via experiment to be 0.61 (section 2.6.3 of text). E.g., we can define the contraction coefficient

$$C_c = \frac{h}{d} = 0.61$$

for this flow as long as the constant head tanks depth is at least $5d$. It is left to the student teams to develop a relationship between the head tank depth and the flow depth just downstream from the sluice gate using the Bernoulli equation, conservation of mass and Equation 3.3. This relationship should be included in your lab report.

**Hydraulic Jump**

If the Bernoulli equation were valid across the hydraulic jump we would expect energy to be conserved and hence the energy grade line elevation to be constant across the jump. If this were the case, given the considerably greater flow depth downstream of the hydraulic jump and resultant slower velocities we would expect the actual water depth to approach the depth of the constant head tank. If this were the case water would be pouring over the sides of our flume! Luckily significant energy is dissipated through the hydraulic jump and energy is not conserved. What are we to do in order to analyze this flow? Well, we turn to the conservation of linear momentum. If we take the left hand (supercritical) side of the hydraulic jump to be at depth $h_1$ with uniform velocity $V_1$ and the right hand side to be at depth $h_2$ and uniform velocity $V_2$ then the steady state conservation of linear momentum equation reduces simply to the net pressure forces acting on our control volume (the residual of the hydrostatic pressures acting on the supercritical and subcritical faces of the control surface) being balanced by the net flux of linear momentum across the control surfaces. Hence we have

$$\frac{1}{2} \rho gb\left(h_1^2 - h_2^2\right) = \dot{m}(V_2 - V_1) = \rho Q(V_2 - V_1) = \rho V_1 h b(V_2 - V_1)$$

where $b$ is the channel width. Eq 3.4 reduces to:
We have one equation with two unknowns \((V_2\) and \(h_2\)). We enforce conservation of mass as our second equation. Therefore

\[
Q = V_1 h_1 b = V_2 h_2 b \Rightarrow V_2 = \frac{h_1}{h_2} V_1
\]

Substituting the result in Eq 3.6 into Eq 3.5 we arrive at:

\[
\frac{1}{2} (h_1^2 - h_2^2) = \frac{V_2^2 h_1}{g} \left( \frac{h_1}{h_2} - 1 \right)
\]

Rearranging and solving for the ratio \(h_2/h_1\) we have:

\[
\left( 1 - \frac{h_2}{h_1} \right) \left( 1 + \frac{h_2}{h_1} \right) = 2 \frac{h_1}{h_2} \frac{V_1^2}{g h_1} \left( 1 - \frac{h_2}{h_1} \right)
\]

Thus we have our first solution – the trivial solution given by \(h_2 = h_1\) – e.g. no jump, the flow remains unchanged. Discarding this solution Eq 3.8 becomes

\[
\left( \frac{h_2}{h_1} \right)^2 + \frac{h_2}{h_1} - 2 \frac{V_1^2}{g h_1} = 0 = \left( \frac{h_2}{h_1} \right)^2 + \frac{h_2}{h_1} - 2Fr^2
\]

Solving this quadratic equation we have

\[
\frac{h_2}{h_1} = \frac{-1 \pm \sqrt{1 + 8Fr^2}}{2}
\]

but clearly the ratio \(h_2/h_1 > 0\) and since \(Fr > 1\) we find the only valid result is

\[
\frac{h_2}{h_1} = \frac{-1 + \sqrt{1 + 8(Fr_1)^2}}{2}
\]

where \(Fr_1\) is the Froude number based on the supercritical conditions. Eq 3.10 is the relationship we have been looking for to give us the flow depth downstream of a hydraulic jump.

**Experimental Apparatus**

The experiment will be conducted in the constant head flume (Figure 2). This flume, as described already above, consists of a constant head inlet section with a variable opening sluice gate at its outlet. The test section is constructed of Plexiglas and terminates in a weir. This weir will force the outlet flow to be supercritical which, as discussed above, insures that surface wave energy will be forced out of the test section and cannot propagate back into the test section. In this lab you will need to measure the energy grade line and the hydraulic grade line. A Pitot tube (note not a Pitot-static tube) is available to measure the stagnation head. You can use this
information, along with the local elevation of the Pitot tube above the flume bed \((z \text{ [cm]})\), to determine the energy grade line. The hydraulic grade line is simply the static head, which is exactly the local elevation of the water surface \((h \text{ [cm]})\). Hence you can use a ruler and simply measure the free surface elevation. The only problem you may encounter with this method is that the surface is wavy and irregular making it challenging to get ruler based measurements with accuracies of millimeters – do you best and think about the average depth, that is what you want.

**Experimental Methods – Green Tank pictured in Figure 1**

1. Fill the tank by filling the white storage tank next to it with the hose. *Ensure that the hose is clamped tightly to the tank* (it can and will spray the entire room). Turn it on using the valve on the pillar behind computer B. Fill the tank until the water depth in the testing tank is 20 cm (the testing tank will fill as you fill the white tank) - read on the ruler fixed to the side of the tank opposite the white tank at about the 3 m downstream of the sluicegate mark.

2. Adjust the sluice gate opening to about 1.5-2.5cm (read the ruler fixed to the gate housing).

3. Turn on the pump (*only when there is water in the tank*) using the switch facing the windows on the pillar directly in front of the key card access door (it will be marked for the lab).

4. After 1-2 minutes (to allow the hydraulic jump to reach steady state) adjust the sluice again so that the beginning of the jump is between 1 to 2 m from the sluice and there is a constant head (it should be open between 1.5 and 2.5 cm). **NEVER CLOSE THE GATE ALL THE WAY DOWN** (there is a stop that should not even allow you to try!). You want to optimize the sluice gate opening such that you get a bit of flow in the overflow.
pipe and the hydraulic jump is located 1 to 2 meters downstream from the sluice gate.
You have achieved constant head if the overflow pipe (PVC pipe that enters the top of the
white storage tank and exits from the constant head tank) is always draining a little bit of
water. *Feel free to climb up and look inside the head box to see how we created the
constant head system – it is very straightforward.* You can also see the water level in a
clear tube attached to a static tap on the side of the constant head tank.

5. The Pitot tube has been set up with a 200 kPa pressure transducer mounted at the top of
the tube. As usual, you need to zero the system before taking the data. Hold the Pitot
tube out of the water (with transducer attached). Ensure that the tube is vertical, then use
the *Easy Data* software to zero it.

6. Once the sluice has been adjusted and the water flow has reached a steady state, begin to
take your measurements. The Pitot tubes are mounted on a simple vertical positioning
system that allows you to raise and lower the Pitot tube by loosening a clamp.

7. Take measurements for the positions listed below and in the “Data Collection/Analysis”
section (*hint*: Use the Appendix to record your data). Use the *Easy Data* software
(shortcut on desktop) to collect the data. Make sure the software is reading pressure for a
200kPa pressure transducer, you are plugged into the correct port, and you are logging to
your group’s data folder. For each measurement you are required to report the average of
pressure value from at least 20 seconds of measurement.

8. After you have taken all of your pressure measurements, turn off the pump at the same
switch that you used to turn it on.

9. Make sure that you measure the dimensions of the tank.

**Data Collection/Analysis**

Your goal is to plot the energy grade line and hydraulic grade line for the flow you establish.
You can measure the energy grade line using a combination of the Pitot tube and a ruler to get
the elevation of the Pitot tube above the bed. Use the Pitot tube to measure the energy grade line
at a minimum of:

- Just under the sluice gate (stick the tube under the gate as far as you can get it to go)
- At the beginning (where the free surface first becomes horizontal), middle, and end of the
  supercritical flow region (3 points minimum), as high up in the water column as you can be
  insured that the Pitot tube is always under water.
- Two points within the hydraulic jump. These won’t be at the same height.
- At the beginning, middle, and end of the subcritical flow region (3 points minimum), again,
  just enough below the free surface to ensure that the Pitot tube is always submerged. Make
  sure that all three are at the same level.

Record the hydraulic grade line at each of these sections as well. Note there is a static tap on the
constant head box near the adjustment for the sluice gate. You can get the elevation of the
constant head condition, relative to the flume test section bed, from this static tap.
You need to measure the flume geometry including the sluice gate opening, $d_s$, the flume width, $b$, and the height of the weir at the outlet of the test section, $d_w$.

Finally, at the downstream end of the supercritical section measure the velocity using the Pitot tube at five different vertical elevations. You should span from the tube resting on the bed (estimate the distance to the center of the Pitot tube orifice), and then just a few millimeters above the bed, then a few more millimeters above the bed, then at about half depth, ¾ depth and just below full depth.

**Laboratory Report**

Submit a report that includes the following

1. Develop a relationship between the constant head tank depth and the flow depth just down stream from the sluice gate. Determine the predicted flow rate and velocity just downstream of the sluice gate (supercritical).
2. Based on the linear momentum analysis above, an estimate of the downstream (sub-critical) flow depth along with your measured value of this depth. Include an estimate of the downstream velocity too, what conservation equation did you have to use?
3. Create a table of your Pitot tube determined velocities, flow rates, and Froude numbers in the supercritical and subcritical regions of the flow. How do these compare to the numbers predicted by the conservation equations? Discuss.
4. Turn in a plot showing the energy and hydraulic grade lines for the flow.
5. Make waves in both the supercritical and subcritical region of the flow, describe their propagation in the along-flume direction.
6. What is the head loss through the hydraulic jump?
7. Can you detect any head loss in the supercritical and/or subcritical reaches from your measurements? Can you detect any head loss through the sluice gate?
8. Where are the regions that the Bernoulli equation is valid for this flow?
9. Plot the velocity profile you measured in the downstream supercritical region of the flow along with a line showing the averaged measured velocity (integrate your profile, so not take the mean of the five readings, why?) and the predicted value found in question 1. Discuss the differences.

**Note:** When working with the Pitot tube before collecting data for your 20+ second averages spend a moment and ensure that the Pitot tube is directed exactly upstream. You can look at the pressure readings and seek to maximize the recorded values, as the pressure will be maximum when the tube is oriented exactly upstream.
### Appendix

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