Clearly, if \( b \) is doubled, the flow rate \( Q \) increases by a factor of \( 2^4 = 16 \). \text{Ans.}

5.19 The period of oscillation \( T \) of a water surface wave is assumed to be a function of density \( \rho \), wavelength \( \lambda \), depth \( h \), gravity \( g \), and surface tension \( Y \). Rewrite this relationship in dimensionless form. What results if \( Y \) is negligible?

\text{Solution:} Establish the variables and their dimensions:

\[
T = \text{fcn}(\rho, \lambda, h, g, Y)
\]

\[
\{T\} \quad \{\text{M/}\text{L}^3\} \quad \{\text{L}\} \quad \{\text{L}\} \quad \{\text{L/T}^2\} \quad \{\text{M/T}^2\}
\]

Then \( n = 6 \) and \( j = 3 \), hence we expect \( n - j = 6 - 3 = 3 \) Pi groups, capable of various arrangements and selected by the writer as follows:

Typical final result: \( T(g/\lambda)^{1/2} = \text{fcn} \left( \frac{h}{\lambda}, \frac{Y}{\rho g \lambda^2} \right) \) \text{Ans.}

If \( Y \) is negligible, \( \rho \) drops out also, leaving: \( T(g/\lambda)^{1/2} = \text{fcn} \left( \frac{h}{\lambda} \right) \) \text{Ans.}

P5.20 A fixed cylinder of diameter \( D \) and length \( L \), immersed in a stream flowing normal to its axis at velocity \( U \), will experience zero average lift. However, if the cylinder is rotating at angular velocity \( \Omega \), a lift force \( F \) will arise. The fluid density \( \rho \) is important, but viscosity is secondary and can be neglected. Formulate this lift behavior as a dimensionless function.

\text{Solution:} No suggestion was given for the repeating variables, but for this type of problem (force coefficient, lift coefficient), we normally choose \((\rho, U, D)\) for the task. List the dimensions:

\[
\begin{array}{ccccccc}
D & L & U & \Omega & F & \rho \\
\{\text{L}\} & \{\text{L}\} & \{\text{LT}^{-1}\} & \{\text{T}^{-1}\} & \{\text{MLT}^{-2}\} & \{\text{ML}^{-3}\}
\end{array}
\]
Solution: Establish the variables and their dimensions:

\[
F = fcn( L, V, D, \alpha, \rho, \mu, a )
\]

\[\{ML/T^2\} \quad \{L\} \quad \{L/T\} \quad \{L\} \quad \{1\} \quad \{M/L^3\} \quad \{M/LT\} \quad \{L/T\}\]

Then \(n = 8\) and \(j = 3\), hence we expect \(n - j = 8 - 3 = 5\) Pi groups. The matrix is

\[
\begin{array}{cccccccc}
F & L & V & D & \alpha & \rho & \mu & a \\
M: & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
L: & 1 & 1 & 1 & 1 & 0 & -3 & -1 & 1 \\
T: & -2 & 0 & -1 & 0 & 0 & 0 & -1 & -1 \\
\end{array}
\]

The rank of this matrix is indeed three, hence there are exactly 5 Pi groups. The writer chooses:

Typical final result: \(\frac{F}{\rho V^2 L^2} = fcn\left( \alpha, \frac{\rho V L}{\mu}, L, V \right)\) \(Ans.\)

P5.25 The thrust \(F\) of a propeller is generally thought to be a function of its diameter \(D\) and angular velocity \(\Omega\), the forward speed \(V\), and the density \(\rho\) and viscosity \(\mu\) of the fluid. Rewrite this relationship as a dimensionless function.

Solution: Write out the function with the various dimensions underneath:

\[
F = fcn( D, \Omega, V, \rho, \mu )
\]

\[\{ML/T^2\} \quad \{L\} \quad \{1/T\} \quad \{L/T\} \quad \{M / L^3\} \quad \{M / LT\}\]

There are 6 variables and 3 primary dimensions (MLT), and we quickly see that \(j = 3\), because \((\rho, V, D)\) cannot form a pi group among themselves. Use the pi theorem to find the three pi’s:
\[\Pi_1 = \rho^a V^b D^c F; \text{ Solve for } a = -1, b = -2, c = -2. \text{ Thus } \Pi_1 = \frac{F}{\rho V^2 D^2}\]

\[\Pi_2 = \rho^a V^b D^c \Omega; \text{ Solve for } a = 0, b = -1, c = 1. \text{ Thus } \Pi_2 = \frac{\Omega D}{V}\]

\[\Pi_3 = \rho^a V^b D^c \mu; \text{ Solve for } a = -1, b = -1, c = -1. \text{ Thus } \Pi_3 = \frac{\mu}{\rho V D}\]

Thus one of many forms of the final desired dimensionless function is

\[\frac{F}{\rho V^2 D^2} = \text{fcn} \left( \frac{\Omega D}{V}, \frac{\mu}{\rho V D} \right) \quad \text{Ans.}\]

5.26 A pendulum has an oscillation period \(T\) which is assumed to depend upon its length \(L\), bob mass \(m\), angle of swing \(\theta\), and the acceleration of gravity. A pendulum 1 m long, with a bob mass of 200 g, is tested on earth and found to have a period of 2.04 s when swinging at 20°. (a) What is its period when it swings at 45°? A similarly constructed pendulum, with \(L = 30\) cm and \(m = 100\) g, is to swing on the moon \((g = 1.62 \text{ m/s}^2)\) at \(\theta = 20^\circ\). (b) What will be its period?

Solution: First establish the variables and their dimensions so that we can do the numbers:

\[T = \text{fcn}(L, m, g, \theta)\]

\[\{T\} \quad \{L\} \quad \{M\} \quad \{L/T^2\} \quad \{1\}\]

Then \(n = 5\) and \(j = 3\), hence we expect \(n - j = 5 - 3 = 2\) \(\Pi\) groups. They are unique:

\[T \sqrt{\frac{g}{L}} = \text{fcn}(\theta) \quad \text{(mass drops out for dimensional reasons)}\]

(a) If we change the angle to 45°, this changes \(\Pi_2\), hence we lose dynamic similarity and do not know the new period. More testing is required. \(\text{Ans. (a)}\)

(b) If we swing the pendulum on the moon at the same 20°, we may use similarity:

\[T_1 \left( \frac{g}{L_1} \right)^{1/2} = (2.04 \text{ s}) \left( \frac{9.81 \text{ m/s}^2}{1.0 \text{ m}} \right)^{1/2} = 6.39 = T_2 \left( \frac{1.62 \text{ m/s}^2}{0.3 \text{ m}} \right)^{1/2},\]

or: \(T_2 = 2.75 \text{ s}\) \(\text{Ans. (b)}\)
5.27 In studying sand transport by ocean waves, A. Shields in 1936 postulated that the bottom shear stress $\tau$ required to move particles depends upon gravity $g$, particle size $d$ and density $\rho_p$, and water density $\rho$ and viscosity $\mu$. Rewrite this in terms of dimensionless groups (which led to the Shields Diagram in 1936).

**Solution:** There are six variables ($\tau$, $g$, $d$, $\rho_p$, $\rho$, $\mu$) and three dimensions (M, L, T), hence we expect $n - j = 6 - 3 = 3$ Pi groups. The author used ($\rho$, $g$, $d$) as repeating variables:

$$\frac{\tau}{\rho g d} = \text{fcn} \left( \frac{\rho^{1/2} d^{3/2}}{\mu}, \frac{\rho_p}{\rho} \right) \quad \text{Ans.}$$

The shear parameter used by Shields himself was based on net weight: $\tau/[(\rho_p - \rho)gd]$.

5.28 A simply supported beam of diameter $D$, length $L$, and modulus of elasticity $E$ is subjected to a fluid crossflow of velocity $V$, density $\rho$, and viscosity $\mu$. Its center deflection $\delta$ is assumed to be a function of all these variables. (a) Rewrite this proposed function in dimensionless form. (b) Suppose it is known that $\delta$ is independent of $\mu$, inversely proportional to $E$, and dependent only upon $\rho V^2$, not $\rho$ and $V$ separately. Simplify the dimensionless function accordingly.

**Solution:** Establish the variables and their dimensions:

$$\delta = \text{fcn}(\rho , D , L , E , V , \mu)$$

{L} {M/L^3} {L} {L} {M/LT^2} {L/T} {M/LT}

Then $n = 7$ and $j = 3$, hence we expect $n - j = 7 - 3 = 4$ Pi groups, capable of various arrangements and selected by the writer, as follows (a):

$$\frac{\delta}{L} = \text{fcn} \left( \frac{L}{D}, \frac{\rho V D}{\mu}, \frac{E}{\rho V^2} \right) \quad \text{Ans. (a)}$$

(b) If $\mu$ is unimportant and $\delta$ proportional to $E^{-1}$, then the Reynolds number ($\rho V D/\mu$) drops out, and we have already cleverly combined $E$ with $\rho V^2$, which we can now slip out and turn upside down:

If $\mu$ drops out and $\delta \propto \frac{1}{E}$, then $\delta = \frac{\rho V^2}{E} \text{fcn} \left( \frac{L}{D} \right)$,

or: $\frac{\delta E}{\rho V^2 L} = \text{fcn} \left( \frac{L}{D} \right) \quad \text{Ans. (b)}$

5.29 When fluid in a pipe is accelerated linearly from rest, it begins as laminar flow and then undergoes transition to turbulence at a time $t_{tr}$ which depends upon the pipe diameter $D$, fluid acceleration $a$, density $\rho$, and viscosity $\mu$. Arrange this into a dimensionless relation between $t_{tr}$ and $D$. 

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**Solutions Manual • Fluid Mechanics, Seventh Edition**
(0.4125)(246)(3.24)/(1.5E-5) = 21,920,000, or log(Re_p) = 7.34. The log-log plot and extrapolation would look like this:

![Log-log plot](image)

You can see that it is a long way out from those four closely packed model points to a Reynolds number of 21,920,000. Uncertainty is high. The model curve-fit \( C_D = 0.82/Re^{0.18} \) can be used to estimate \( C_D(\text{prototype}) = 0.82/(21,920,000)^{0.18} = 0.039 \). Our rather uncertain estimate for the drag of the full-scale aircraft is thus

\[
\text{Full-scale drag} \approx C_D(\rho_p/2)V_p^2A_p = (0.039)(0.4125/2)(246)^2(90.7) \approx 44,000 \text{ N} \approx 10,000 \text{ lbf} \quad \text{Ans.}
\]

5.73 The power \( P \) generated by a certain windmill design depends upon its diameter \( D \), the air density \( \rho \), the wind velocity \( V \), the rotation rate \( \Omega \), and the number of blades \( n \).

(a) Write this relationship in dimensionless form. A model windmill, of diameter 50 cm, develops 2.7 kW at sea level when \( V = 40 \text{ m/s} \) and when rotating at 4800 rev/min. (b) What power will be developed by a geometrically and dynamically similar prototype, of diameter 5 m, in winds of 12 m/s at 2000 m standard altitude? (c) What is the appropriate rotation rate of the prototype?
Solution: (a) For the function \( P = \text{fcn}(D, \rho, V, \Omega, n) \) the appropriate dimensions are \( \{P\} = \{ML^2T^{-3}\}, \{D\} = \{L\}, \{\rho\} = \{ML^{-3}\}, \{V\} = \{L/T\}, \{\Omega\} = \{T^{-1}\}, \) and \( \{n\} = \{1\} \). Using \((D, \rho, V)\) as repeating variables, we obtain the desired dimensionless function:

\[
\frac{P}{\rho D^2 V^3} = \text{fcn} \left( \frac{\Omega D}{V}, n \right) \quad \text{Ans. (a)}
\]

(c) “Geometrically similar” requires that \( n \) is the same for both windmills. For “dynamic similarity,” the advance ratio \((\Omega D/V)\) must be the same:

\[
\left( \frac{\Omega D}{V} \right)_{\text{model}} = \left( \frac{4800 \text{r/min}(0.5 \text{ m})}{40 \text{ m/s}} \right) = 1.0 = \left( \frac{\Omega D}{V} \right)_{\text{proto}} = \frac{\Omega_{\text{proto}}(5 \text{ m})}{12 \text{ m/s}},
\]

or: \( \Omega_{\text{proto}} = 144 \text{ rev/min} \quad \text{Ans. (c)} \)

(b) At 2000 m altitude, \( \rho = 1.0067 \text{ kg/m}^3 \). At sea level, \( \rho = 1.2255 \text{ kg/m}^3 \). Since \( \Omega D/V \) and \( n \) are the same, it follows that the power coefficients equal for model and prototype:

\[
\frac{P}{\rho D^2 V^3} = \frac{2700W}{(1.2255)(0.5)^2 (40)^3} = 0.138 = \frac{P_{\text{proto}}}{(1.0067)(5)^2 (12)^3},
\]

solve \( P_{\text{proto}} = 5990 \text{ W} \approx 6 \text{ kW} \quad \text{Ans. (b)} \)

5.74 A one-tenth-scale model of a supersonic wing tested at 700 m/s in air at 20°C and 1 atm shows a pitching moment of 0.25 kN·m. If Reynolds-number effects are negligible, what will the pitching moment of the prototype wing be flying at the same Mach number at 8-km standard altitude?

Solution: If Reynolds number is unimportant, then the dimensionless moment coefficient \( M/(\rho V^2 L^3) \) must be a function only of the Mach number, \( Ma = V/a \). For sea-level air, take \( \rho = 1.225 \text{ kg/m}^3 \) and sound speed \( a = 340 \text{ m/s} \). For air at 8000-m standard altitude (Table A-6), take \( \rho = 0.525 \text{ kg/m}^3 \) and sound speed \( a = 308 \text{ m/s} \). Then

\[
Ma_m = \frac{V_m}{a_m} = \frac{700}{340} = 2.06 = Ma_p = \frac{V_p}{a_p}, \quad \text{solve for} \quad V_p = 634 \text{ m/s}
\]

Then

\[
M_p = M_m \left( \frac{\rho_p V_p^2 L_p^3}{\rho_m V_m^2 L_m^3} \right) = 0.25 \left( \frac{0.525}{1.225} \right) \left( \frac{634}{700} \right)^2 \left( \frac{10}{1} \right)^3 \approx 88 \text{ kN·m} \quad \text{Ans.} \]
According to the web site *USGS Daily Water Data for the Nation*, the mean flow rate in the New River near Hinton, WV is 10,100 ft$^3$/s. If the hydraulic model in Fig. 5.9 is to match this condition with Froude number scaling, what is the proper model flow rate?

**Solution:** For Froude scaling, the volume flow rate is a blend of velocity and length terms:

$$\frac{Q_m}{Q_p} = \frac{V_m A_m}{V_p A_p} = \sqrt{\frac{L_m}{L_p}} \left(\frac{L_m}{L_p}\right)^2 = \left(\frac{L_m}{L_p}\right)^{5/2} \quad \text{or} \quad \alpha^{5/2}$$

Fig. 5.9: $\alpha = 1:65$; \therefore $Q_{model} = \left(10100 \frac{ft^3}{s}\right)\left(\frac{1}{65}\right)^{5/2} = 0.30 \frac{ft^3}{s}$ \textit{Ans.}

A 2-ft-long model of a ship is tested in a freshwater tow tank. The measured drag may be split into “friction” drag (Reynolds scaling) and “wave” drag (Froude scaling). The model data are as follows:

- Tow speed, ft/s: 0.8 1.6 2.4 3.2 4.0 4.8
- Friction drag, lbf: 0.016 0.057 0.122 0.208 0.315 0.441
- Wave drag, lbf: 0.002 0.021 0.083 0.253 0.509 0.697

The prototype ship is 150 ft long. Estimate its total drag when cruising at 15 kn in seawater at 20°C.

**Solution:** For fresh water at 20°C, take $\rho = 1.94$ slug/ft$^3$, $\mu = 2.09E-5$ slug/ft⋅s. Then evaluate the Reynolds numbers and the Froude numbers and respective force coefficients:

- $V_m$, ft/s: 0.8 1.6 2.4 3.2 4.0 4.8
- $\text{Re}_m = \frac{V_m L_m}{\nu}$: 143000 297000 446000 594000 743000 892000
- $\text{CF}_{\text{friction}}$: 0.00322 0.00287 0.00273 0.00261 0.00254 0.00247
- $\text{Fr}_m = \frac{V_m}{\sqrt{gL_m}}$: 0.099 0.199 0.299 0.399 0.498 0.598
- $\text{CF}_{\text{wave}}$: 0.00040 0.00106 0.00186 0.00318 0.00410 0.00390
For seawater, take $\rho = 1.99 \text{ slug/ft}^3$, $\mu = 2.23 \times 10^{-5} \text{ slug/ft \cdot s}$. With $L_p = 150 \text{ ft}$ and $V_p = 15 \text{ knots} = 25.3 \text{ ft/s}$, evaluate

$$Re_{\text{proto}} = \frac{\rho_p V_p L_p}{\mu_p} = \frac{1.99(25.3)(150)}{2.23 \times 10^{-5}} \approx 3.39 \times 10^8; \quad Fr_p = \frac{25.3}{[32.2(150)]^{1/2}} \approx 0.364$$

For $Fr \approx 0.364$, interpolate to $C_{F,\text{wave}} \approx 0.0027$

Thus we can immediately estimate $F_{\text{wave}} \approx 0.0027(1.99)(25.3)^2(150)^2 \approx 77000 \text{ lbf}$. However, as mentioned in Fig. 5.8 of the text, $Re_p$ is far outside the range of the friction force data, therefore we must extrapolate as best we can. A power-law curve-fit is

$$C_{F,\text{friction}} = \frac{0.0178}{Re^{0.144}}, \quad \text{hence} \quad C_{F,\text{proto}} \approx \frac{0.0178}{(3.39 \times 10^8)^{0.144}} \approx 0.00105$$

Thus $F_{\text{friction}} \approx 0.00105(1.99)(25.3)^2(150)^2 \approx 30000 \text{ lbf}$. Therefore $F_{\text{total}} \approx 107000 \text{ lbf}$. \textit{Ans.}

5.77 A dam spillway is to be tested by using Froude scaling with a one-thirtieth-scale model. The model flow has an average velocity of 0.6 m/s and a volume flow of 0.05 m$^3$/s. What will the velocity and flow of the prototype be? If the measured force on a certain part of the model is 1.5 N, what will the corresponding force on the prototype be?

\textbf{Solution:} Given $\alpha = L_m/L_p = 1/30$, Froude scaling requires that

$$V_p = \frac{V_m}{\sqrt{\alpha}} = \frac{0.6}{(1/30)^{1/2}} = 3.3 \text{ m/s}; \quad Q_p = \frac{Q_m}{\alpha^{5/2}} = \frac{0.05}{(1/30)^{5/2}} = 246 \text{ m}^3/\text{s} \quad \text{Ans. (a)}$$

The force scales in similar manner, assuming that the density remains constant (water):

$$F_p = F_m \left( \frac{\rho_p}{\rho_m} \right) \left( \frac{V_p}{V_m} \right)^2 \left( \frac{L_p}{L_m} \right)^2 = F_m (1) \left( \frac{1}{\sqrt{\alpha}} \right)^2 \left( \frac{1}{\alpha} \right)^2 = (1.5 \text{ N})(30)^3 \approx 40500 \text{ N} \quad \text{Ans. (b)}$$

5.78 A prototype spillway has a characteristic velocity of 3 m/s and a characteristic length of 10 m. A small model is constructed by using Froude scaling. What is the minimum scale ratio of the model which will ensure that its minimum Weber number is 100? Both flows use water at 20°C.

\textbf{Solution:} For water at 20°C, $\rho = 998 \text{ kg/m}^3$ and $Y = 0.073 \text{ N/m}$, for both model and prototype. Evaluate the Weber number of the prototype:
For seawater, take \( \rho = 1.99 \text{ slug/ft}^3 \), \( \mu = 2.23 \times 10^{-5} \text{ slug/ft} \cdot \text{s} \). With \( L_p = 150 \text{ ft} \) and \( V_p = 15 \text{ knots} = 25.3 \text{ ft/s} \), evaluate

\[
\text{Re}_{\text{proto}} = \frac{\rho_p V_p L_p}{\mu_p} = \frac{1.99(25.3)(150)}{2.23 \times 10^{-5}} \approx 3.39 \times 10^8; \quad \text{Fr}_p = \sqrt[2]{\frac{25.3}{[32.2(150)]^{1/2}}} \approx 0.364
\]

For \( \text{Fr} = 0.364 \), interpolate to \( C_{\text{F, wave}} = 0.0027 \). Thus we can immediately estimate \( F_{\text{wave}} \approx 0.0027(1.99)(25.3) = 77000 \text{ lbf} \). However, as mentioned in Fig. 5.8 of the text, \( \text{Re} \) is far outside the range of the friction force data, therefore we must extrapolate as best we can. A power-law curve-fit is

\[
C_{\text{F, friction}} = \frac{0.0178}{\text{Re}^{0.144}}, \quad \text{hence} \quad C_{\text{F, proto}} = \frac{0.0178}{(3.39 \times 10^8)^{0.144}} \approx 0.00105
\]

Thus \( F_{\text{friction}} \approx 0.00105(1.99)(25.3)^2(150)^2 = 30000 \text{ lbf} \).

\( F_{\text{total}} \approx 107000 \text{ lbf} \). \text{Ans.}

5.77 A dam spillway is to be tested by using Froude scaling with a one-thirtieth-scale model. The model flow has an average velocity of 0.6 m/s and a volume flow of 0.05 m\(^3\)/s. What will the velocity and flow of the prototype be? If the measured force on a certain part of the model is 1.5 N, what will the corresponding force on the prototype be?

**Solution:** Given \( \alpha = L_m/L_p = 1/30 \), Froude scaling requires that

\[
V_p = \frac{V_m}{\sqrt{\alpha}} = \frac{0.6}{(1/30)^{1/2}} = 3.3 \text{ m/s}; \quad Q_p = \frac{Q_m}{\alpha^{5/2}} = \frac{0.05}{(1/30)^{5/2}} = 246 \text{ m}^3/\text{s} \quad \text{Ans. (a)}
\]

The force scales in similar manner, assuming that the density remains constant (water):

\[
F_p = F_m \left( \frac{\rho_p}{\rho_m} \right) \left( \frac{V_p}{V_m} \right)^2 \left( \frac{L_p}{L_m} \right)^2 = F_m \left( \frac{1}{\sqrt{\alpha}} \right)^2 \left( \frac{1}{\alpha} \right)^2 = (1.5 \text{ N})(30)^3 = 40500 \text{ N} \quad \text{Ans. (b)}
\]

5.78 A prototype spillway has a characteristic velocity of 3 m/s and a characteristic length of 10 m. A small model is constructed by using Froude scaling. What is the minimum scale ratio of the model which will ensure that its minimum Weber number is 100? Both flows use water at 20\(^\circ\)C.

**Solution:** For water at 20\(^\circ\)C, \( \rho = 998 \text{ kg/m}^3 \) and \( Y = 0.073 \text{ N/m} \), for both model and prototype. Evaluate the Weber number of the prototype:
\[ \text{We}_p = \frac{\rho_p V_p^2 L_p}{Y_p} = \frac{998(3.0)^2(10.0)}{0.073} \approx 1.23 \times 10^6; \text{ for Froude scaling,} \]

\[ \frac{\text{We}_m}{\text{We}_p} = \frac{\rho_m}{\rho_p} \left( \frac{V_m}{V_p} \right)^2 \left( \frac{L_m}{L_p} \right) \left( \frac{Y_p}{Y_m} \right) = (1)(\sqrt{\alpha})^2(\alpha)(1) = \alpha^2 = \frac{100}{1.23 \times 10^6} \text{ if } \alpha = 0.0090 \]

Thus the model Weber number will be \(\geq 100\) if \(\alpha = \frac{L_m}{L_p} \geq 0.0090 = 1/111\). Ans.

5.79 An East Coast estuary has a tidal period of 12.42 h (the semidiurnal lunar tide) and tidal currents of approximately 80 cm/s. If a one-five-hundredth-scale model is constructed with tides driven by a pump and storage apparatus, what should the period of the model tides be and what model current speeds are expected?

Solution: Given \(T_p = 12.42\) hr, \(V_p = 80\) cm/s, and \(\alpha = \frac{L_m}{L_p} = 1/500\). Then:

Froude scaling: \(T_m = T_p \sqrt{\alpha} = \frac{12.42}{\sqrt{500}} = 0.555\) hr \(\approx 33\) min Ans. (a)

\(V_m = V_p \sqrt{\alpha} = 80/\sqrt{(500)} = 3.6\) cm/s Ans. (b)

5.80 A prototype ship is 35 m long and designed to cruise at 11 m/s (about 21 kn). Its drag is to be simulated by a 1-m-long model pulled in a tow tank. For Froude scaling find (a) the tow speed, (b) the ratio of prototype to model drag, and (c) the ratio of prototype to model power.

Solution: Given \(\alpha = 1/35\), then Froude scaling determines everything:

\(V_{\text{tow}} = V_m = V_p \sqrt{\alpha} = 11/\sqrt{(35)} = 1.86\) m/s

\(\frac{F_m}{F_p} = \left( \frac{V_m}{V_p} \right)^2 \left( \frac{L_m}{L_p} \right)^2 = (\sqrt{\alpha})^2(\alpha)^2 = \alpha^3 = (1/35)^3 = \frac{1}{42900}\) Ans.

\(\frac{P_m}{P_p} = (F_m/F_p)(V_m/V_p) = \alpha^2(\sqrt{\alpha}) = \alpha^{3.5} = 1/35^{3.5} = \frac{1}{254000}\)

5.81 An airplane, of overall length 55 ft, is designed to fly at 680 m/s at 8000-m standard altitude. A one-thirtieth-scale model is to be tested in a pressurized helium wind tunnel at 20°C. What is the appropriate tunnel pressure in atm? Even at this (high) pressure, exact dynamic similarity is not achieved. Why?
1) \( Q \rightarrow \) Bernoulli from 1 to 2

\[
\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2
\]

\( z_1 = z_2, \ p_1 = 0, \ p_2 = \gamma h, \ V_2 = 0 \)

Stagnation streamline

\[
\therefore V_1^2 = 2gh \Rightarrow V_1 = \sqrt{2gh}
\]

Assume 1-D constant C.U., cons. linear momentum in x-direction.

\[
\sum F_{ext} = mV_{in} - mV_{out} \quad \Rightarrow \quad \boxed{\text{Reaction force}}
\]

\[
R_x = -mV_{in} = -\rho QV_1 = -\rho Q\sqrt{2gh}
\]

Resultant force \( F_x = -R_x = \frac{\rho Q\sqrt{2gh}}{S} \)

\[
\therefore F_x = 999 \text{ kg} \cdot \text{m/s} \cdot 1 \text{ m}^2 \cdot \sqrt{2 \cdot 9.81 \text{ m/s}^2 \cdot 4 \text{ cm}} = 100 \text{ kg m/s}^2
\]

\[
\therefore F_x = 100 \text{ N}
\]

2) \( \sqrt{D} \leq Q \)

\( \rightarrow D \)

\( \sigma = \rho \)

\( \theta = \alpha \)

\( \delta \rightarrow \alpha \)

i) \( d = f(\sigma, D, Q, \alpha, \rho, g) \)

Surface tension

ii) 7 variables, 3 dim's \( \Rightarrow \) 4 \( \alpha \) 's

\[
\alpha_1 = \frac{\sigma}{D}, \quad \alpha_2 = \frac{\sigma}{D} = \frac{\rho Q D}{\rho D D^2} = \frac{4 Q \sigma}{\pi D}
\]

or \( \alpha_2 = \frac{\rho Q}{\pi D} \nabla \quad \text{Note, I choose } D \text{ not } \alpha \) because I suspect pipe flow \( \text{Re} \) is important but \( \text{either is O.K.} \)

\[
\alpha_3 = \frac{\rho Q}{D^3} \quad \text{or} \quad \alpha_4 = \frac{\rho Q^2}{D^4}
\]

\[
\alpha_4 = \frac{\rho Q^2}{D^4}
\]

\[
\therefore \frac{d}{D} = g(\text{Re, Fr}, \text{We})
\]

\[
\therefore \frac{d}{D} = g \left( \frac{\rho Q}{2.0 \text{ mm}}, \frac{\sigma}{VgD^2}, \frac{\rho Q^2 D}{D^4} \right)
\]

\( \text{c) } \frac{DP}{Dm} = 0.05 \text{ mm} = \frac{1}{40} \text{ cm. } \text{Re} \Rightarrow \frac{Qm}{Dm} = \frac{QD}{DP} \text{ same fluid properties.}
\]

\[
\therefore \frac{Qm}{QD} = 40 \quad \therefore \text{Qm = 40 QD, O.K.}
\]
For \( F_r = \frac{\partial p}{\partial z} = \frac{\partial p}{\partial y} = \frac{\partial p}{\partial \varphi} \),

\[ \frac{\partial p}{\partial \varphi} = \left( \frac{D_n}{D_m} \right)^{1/2} \omega \left( \frac{D_m}{D_n} \right)^{1/2} = 1 \times 10^4 \text{ cm/cm!} \]

3. TOTAL SIMILARITY CANNOT BE ACHIEVED.

\[ \frac{\omega}{R} \quad \text{CONS. ANGULAR MUMM} \]

\[ \frac{1}{R_1} \quad \omega/2 \]

\[ \sum(\sum (T \times n)) = \sum[\sum (T \times n)] \]

\[ \omega = -T_0 \in \text{ PROCEED TURBO OPPOSITE TO } \omega \text{ IN C.W. } \text{-} \text{DIRECTION} \]

\[ \vec{T}_0 = \vec{m}(R_1 V_1 + R_2 V_2) \quad V_1 = \omega R_1 + \text{VELOCITY} \quad V_2 = \omega R_2 + \text{VELOCITY} \]

\[ \text{VELOCITY} = \frac{2 \omega}{R} \quad \frac{R_1^2 + R_2^2}{R_1^2 + R_2^2} \]

\[ m = \rho \omega \quad \frac{T_0}{\omega} = \rho \omega \left[ R_1 \left( \omega R_1 - \frac{2 \omega}{R_1^2} \right) + R_2 \left( \omega R_2 - \frac{2 \omega}{R_2^2} \right) \right] \]

Solving for \( \omega \):

\[ \frac{T_0}{\omega} = \omega \left( R_1^2 + R_2^2 \right) - \frac{2 \omega}{R_1^2} \left( R_1 + R_2 \right) \]

\[ \omega = \frac{2 \omega}{R_1^2 + R_2^2} \left( R_1^2 + R_2^2 \right) - \frac{2 \omega}{R_1^2} \left( R_1 + R_2 \right) \]

\[ \omega = 3.4 \text{ RADS} \left( \frac{1 \text{ RPM}}{2 \pi \text{ RADS}} \right) \left( \frac{60 \text{ S}}{1 \text{ MIN}} \right) \]

\[ \omega = 33 \text{ RPM} \]

4. INFINITE IN Y

\[ \frac{\partial \omega}{\partial y} = 0 \quad \text{-} \text{D GIVEN CONSTANT FROM CONTINUITY!} \]

\[ \frac{2 \omega}{\partial x} + \frac{2 \omega}{\partial x} = 0 \quad \frac{2 \omega}{\partial x} = -2 \frac{u_0}{2} \left[ \frac{u_0}{2} \left( \frac{2}{C^2 x^2} \right) - \frac{2}{C^2 x^2} \right] \]

\[ = -2 \omega \frac{2}{C^2 x^2} - \omega \frac{2}{C^2 x^2} \]

Now integrate in \( z \)

\[ \omega(x,z) = \left( \frac{2}{2C x^{3/2}} \right) \left( \frac{2}{3C^2 X^2} \right) + C(z) \]

BUT NO NORMAL FLOW AT WALL \( z = 0 \) \( C(z) = 0 \)

\[ \omega(x,z) = \left( \frac{2}{2C x^{3/2}} \right) \left( \frac{2}{3C^2 X^2} \right) + C(0) \]

\[ \omega(x,z) = \omega_0 \left( \frac{2}{2C x^{3/2}} \right) \left( \frac{2}{3C^2 X^2} \right) + C(0) \]

\[ \omega(x,z) = \omega_0 \left( \frac{2}{2C x^{3/2}} \right) \left( \frac{2}{3C^2 X^2} \right) \]
1) Since wave drag is dominant we can choose to neglect roughness but ok. to include too.

\[ F_0 = f(L, \beta, M, V, g, \rho) \]

2c) 7 variables - 3 dimensions = 4 IT's

By inspection

\[ \frac{F_0}{\rho L^2 U^2} = g \left( \frac{\rho L U}{\mu}, \frac{\rho L U^2}{\eta}, \frac{V}{\sqrt{g} L} \right) \]

- or - \[ C_0 = g \left( R_e, \text{We}, F_R \right) \]
\[ T_4 = g \frac{Q^2}{2} (b_4 + q^4) = \left[ \frac{L^2}{V^2} \right] \left[ \frac{L^2}{V^2} \right] \left[ \frac{L^2}{V^2} \right] \left[ \frac{L^2}{V^2} \right] = \frac{L^2}{V^2} \]

\[ a_4 = 0 \quad 1 + b_4 + c_4 = -a \quad -c_4 = -2 \quad b_4 + 1 \]

\[ \frac{L_4}{V_4} = \frac{9L}{V_4} \]

\[ F_D \quad \frac{L^2}{V^2} = f \left( \frac{P_4}{L^2}, \frac{V_4}{L^2}, \frac{P_4}{V^2} \right) = f \left( Re, Fr, We \right) \]

(3) \[ \frac{L_4}{V_4} = \frac{9L}{V_4} \]

For \( Re \)
\[ \frac{L_m V_m}{L_p V_p} \quad \Rightarrow \frac{V_m}{V_p} = \frac{L_p}{L_m} = 6 \quad \Rightarrow \frac{L_m}{L_p} = 6 \]

For \( Fr \)
\[ \frac{V_m}{V_p} = \frac{V_m}{V_p} \quad \Rightarrow \frac{V_m}{V_p} = \sqrt{\frac{L_p}{L_m}} = \sqrt{6} \quad \Rightarrow \frac{V_m}{V_p} = \sqrt{6} \]

If we are using the same fluid \( \rho, \mu, \gamma \) will be the same
\[ \Rightarrow \text{can't achieve total similarity.} \]

(4) \( Fr \)-similarity.
\[ \left( \frac{F_D}{L^2 V^2} \right)_m = \left( \frac{V}{V_p} \right)_m \quad \Rightarrow \frac{L_m}{V_m} = \frac{L_p}{V_p} \]

\[ \frac{V_m}{V_p} = \frac{L_p}{L_m} = 6 \]

\[ L_m = 6 \times \frac{1}{2} = 1 \text{ m} \]

\[ \frac{L_m}{L_p} = \frac{1}{6} \]

\[ \frac{F_D}{L^2 V^2} \quad \left( \frac{V}{V_p} \right)_p \quad \Rightarrow \frac{V_m}{V_p} = \frac{V_m}{V_p} \quad \Rightarrow \frac{V_m}{V_p} = \sqrt{6} \]

(1) Geometrically similar
\[ \frac{L_m}{L_p} = \frac{L_m}{L_p} = \frac{1}{6} \]

(2) Kinematic similarity
\[ \left( \frac{V_m}{V_p} \right) = \frac{V_m}{V_p} \]

\[ F_D = \frac{F_D}{F_p} \quad \frac{L_m}{L_p} = \frac{V_m}{V_p} \]

\[ \Rightarrow F_D = \frac{F_D}{F_p} \quad \left( \frac{V_m}{V_p} \right) = \frac{V_m}{V_p} \]

\[ \Rightarrow F_D = \frac{F_D}{F_p} \quad \left( \frac{V_m}{V_p} \right) = \frac{V_m}{V_p} \]

\[ \Rightarrow F_D = \frac{F_D}{F_p} \quad \left( \frac{V_m}{V_p} \right) = \frac{V_m}{V_p} \]

\[ \Rightarrow F_D = \frac{F_D}{F_p} \quad \left( \frac{V_m}{V_p} \right) = \frac{V_m}{V_p} \]

(5) a. The Froude number eliminates |

b. Gravitational acceleration and density are the same |

using the same fluid.