Due 8:00 am Tuesday, February 26, 2002

1) By 8:00 am on Tuesday, February 26 email me (eac20@cornell.edu) a paragraph or so with your thoughts on an interesting case study for your project. You may list more than one idea of you are not sure what you want to do. If you have no idea please write a paragraph or so on what your specific areas of interest in Environmental Fluid Mechanics are. If you have a reasonable idea of who you will work with on the project and what the project will be you may submit a single email for the group but clearly state who is in the group. Groups can be a maximum of three people and should be at least two people.

Due Thursday, March 7, 2002

For the following problems you may work in groups but please list the people you have consulted in doing the problems and for which problems you have consulted with whom. Each person is to hand in his or her own solution.

2) Derive the full solution to the linearized gravity wave problem making the shallow water assumption $kh << 1$. Verify your solution by investigating the limiting condition of the general solution in the course notes (equations 3.59 – 3.63) in the limit $kh << 1$.

3) Re-scale the 2D Navier Stokes equations making the assumption $kh << 1$ but do not make the assumption $ak << 1$. What is the simplest form of the equations you can find? These are the shallow water wave equations and are the most often form of the equations used large scale environmental flows – we will see them again when we work on Cayuga Lake.

Extra Credit: Find a general solution to the shallow water wave equations.

4) Why do waves strike a beach nearly perpendicularly? Your chance to see! At some point in your engineering/physics careers you have likely run into Snell’s law which relates the angle of light rays (remember – light is a wave, albeit non-dispersive!) with the speed of propagation (the index-of-refraction of a material is the ratio of the speed of light in a vacuum to the speed of light in that material, hence it is always greater or equal to 1). As a reminder Snell’s law is written:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

where $n_1$ is the index of refraction of material 1 and $\theta_1$ is the angle of the the wave ray (defined as the perpendicular to lines of constant wave phase – e.g., a wave
front or crest in the water wave case) in material one and the subscript 2 denotes the second material.

(i) First write Snell’s law in a form appropriate for water waves.

(ii) Now, the conservation of wave crests says that wave frequency is a constant. Use this, a starting angle between the wave crest and a straight shoreline of 40°, a constant offshore slope of 3% (isobaths parallel to the straight shoreline) and wave period of 15s to determine the angle between the wave crests and the shoreline when this wave, which starts in deep water, reaches the shoreline. You will need to track angle changes as a function of propagation. I leave it to you to break the path into a reasonable number of points to get a reasonably accurate solution!

(iii) Do you have any concerns about the validity of your solution all the way to the coast? Why? (If you don’t yet answer the next question and see if that changes your perspective).

(iv) Considering two wave rays (the line perpendicular to the wave crest as it propagates on shore) starting from points spaced a fixed distance apart, say s, parallel to the coast, determine an equation for the wave amplitude that accounts for the wave entering shallower water. You may assume that the energy between the two wave rays is constant (e.g., energy travels only along the rays, and does not travel perpendicular to them).

(v) Use your result to determine the wave amplitude and wave slope at each point you used Snell’s law. You may take the initial deep water amplitude to be 2 m.