CEE 3510 ENVIRONMENTAL QUALITY ENGINEERING

STELLA Exercise #1

This exercise will help you, step-by-step, build a simple model with STELLA. You will then use your model to answer questions which will demonstrate some of STELLA's capabilities. You should have completed the "Introduction to STELLA" before attempting this assignment since some necessary information was covered in it. You must have purchased a copy of the STELLA program, or be using one of the copies available for loan at a campus computing facility. The instructions and illustrations given in this exercise assume that STELLA version 10.0.6 is being used. Earlier versions of STELLA can be used, but the steps you will need to take to build a model may vary in some cases.

If you have not already opened-up the STELLA program, please do so. Navigate using the tabs so that you are looking at the blank Model layer of STELLA. A diagram of the model layer showing the layout of the tools (in STELLA v 10.0.6) is shown below.

You will now develop a model for decay of a radioactive material. A property of a radioactive substance is that it is unstable and "decays" by emission of a helium nucleus (alpha particles), electrons (beta particles), and/or photons (gamma radiation). The rate
at which any radioactive isotope decays is unique to that particular material. The decay rate is commonly expressed as a "half-life," which is the amount of time required for one half of the isotope to decay. The half-life and the decay rate constant (i.e., the fraction of radioactive material decaying per unit of time) are interrelated by the following equation:

\[ \text{half life} = \frac{\ln 2}{\text{decay rate constant}} \]

Let's begin a model with STELLA. In STELLA, a "stock" is used to represent a unit of a model that is changing over time. In this situation, the stock will be the amount of radioactive material, which is changing with time.

Select the "stock" from the tool kit (the first item on the top left in STELLA v 10.0.6) by using the mouse to click on the rectangular stock icon. Be sure to release the mouse button once you've clicked. Next, slide your mouse to position the cursor in the middle of the empty tableau (note that the cursor now takes the form of a miniaturized stock). Click once to "deposit" the stock.

Your cursor has automatically changed back to an arrow. The arrow is very useful for selecting model components and tools and moving things around. STELLA has named the stock, "Noname 1". Rename the stock as described below.

If the stock is highlighted (i.e., darkened), simply type in, “SOURCE.” If for some reason, your stock is not highlighted, move the mouse to inside of the stock, and click once. This should highlight the unit and its name. Type the new name. When your name is correct, click on an empty space away from the stock.

At present your stock is undefined, because you have not told STELLA how much source there is. This is the value that STELLA uses to begin the simulation. We will do this in a minute, but for now we will continue to assemble the diagram of the model. Now you should ask, "What factors are changing the amount of source?” The source is not growing, but it is decaying away. We indicate this loss by a Flow out of the stock.
Select the Flow icon (the second icon from the left in the STELLA v 10.0.6 tool kit) by clicking once on it. Bring the icon onto the tableau, and position it anywhere inside of the SOURCE stock. Click-and-hold down the button, then drag the icon out through the right wall of the stock. When you've gone two inches or so, release your click. The flow should be highlighted (If it is not, click once on it.). Type, “DECAY RATE”, to name this outflow.

By making the flow leave the SOURCE stock, you are telling STELLA that the amount of source will decrease somehow (i.e., by radioactive decay). The cloud at the end of your flow is an "infinite" reservoir. We do not care where the SOURCE goes (once it has completely decayed it is no longer radioactive). Thus, it appears as a "cloud." Later in this exercise, you may want to use the flow to interconnect two stocks. To do this, just start the flow in one stock and be sure it ends inside the other. In this case, no cloud should appear.

With radioactive material, there is a constant fraction that decays each year. We can indicate this factor in our model with the “Converter” tool, since the fraction will not be changing over time. Converters in the STELLA program represent constants.

Select the “Converter” -- the third icon from the left in the v 10.0.6 Tool Kit -- by clicking once on it. Position it about an inch or so to the right and below DECAY RATE. Click once to deposit it. If you do not like where the element landed, position the arrow inside it, click-and-hold, slide it to a more desirable location, and then release your click. You can re-position elements in this manner, even if they are attached to other elements. Name this converter, "FRACTION DECAYING”.

The overall decay rate is a function of the fraction decaying and the amount of radioactive material. We use the “Action Connector” (arrow) to indicate these relationships.
Click on the Action Connector - the fourth icon in the v10.0.6 Tool Kit. Then, position the cursor inside FRACTION DECAYING. Click-and-hold the mouse button, and drag the cursor toward the circle portion of DECAY RATE. When the arrow contacts the circle, it will gray. At this point, release your click. You need not draw straight lines when making connections.

Click on the Arrow button again. Position the cursor inside SOURCE. Click-and-hold the mouse button, and drag the cursor to the circle portion of DECAY RATE. After contact, release your click.

Your model should look something like this now:

If your connecting arrows are a bit "wild", you can reposition them by using the hand to click on the little circle at the base of the arrow. Click-and-hold on the little circle, and then move it to a more desirable location on the edge of the converter or stock in question and release your click. The starting location determines the shape of the arrow. Moving the circle is a bit tricky and may take several tries. This completes the Diagram of your model. Now, we need to tell STELLA what this all means.

If you need to stop in the middle of any assignment, please be sure to save the work you have already done by saving it to your computer or, if you are at a campus computing facility, to some form of portable memory (memory stick, etc.). When you save, make sure the destination name is your CEE 3510 memory device, not the hard drive if you are at the engineering library or at a CIT facility. It is good practice to occasionally save your models just in case any problems arise.
Now, let’s tell STELLA more about our model. Click on the **Equation** tab at the left of the screen to move to the equations layer. You should find some text and several ? marks (see the picture shown below). The ?s indicate some terms or relationships are undefined. Your model won’t run as long as any ?s remain.

![Picture of the STELLA Equations view showing undefined terms.](image)

Let’s start by telling STELLA how SOURCE changes over time. Return to the Model view and double click on the DECAY RATE flow. A definition box should open up (if it doesn't, try clicking faster). You should now see a box, labeled "Required Inputs", which lists variables you need to use in the DECAY RATE equation. From this list, click once on FRACTION_DECAYING. It should appear on the right side of the equation (i.e., in the large box). Below the Required Inputs box, there is a row of Operators. You can enter operators into your equations by clicking on them. You can also just use your computer’s keyboard for all entries. Click on the multiplication symbol (*) or type it in. Next, from the list of required inputs, click on SOURCE.

You should now have:
"FRACTION_DECAYING*SOURCE" in the large box which is the right hand side of your equation. The complete equation is:

\[
\text{DECAY RATE} = \text{FRACTION_DECAYING} \times \text{SOURCE}
\]

The units are Curies/year. Type the units in after your
equation inside \{\} brackets. Units must be enclosed in brackets (i.e. \{Curies/year\}) so STELLA doesn’t confuse them with the equation. STELLA also has a units box below your equation, but we will not make use of it in this exercise. When you have the equation correct, the ? next to DECAY_RATE will have gone away in the Equation view. You can close dialog boxes by clicking on the \(\triangleright\) symbol at the middle of the left border.

We need to tell STELLA what the starting value of the SOURCE is. In the Model view double click on the SOURCE icon. Type 900 in the equation box. (Backspacing corrects mistakes.) Define the units to be Curies. Remember the units must be inside \{\} brackets.

[For the curious, the Curie is a unit system used to express the quantity of a radioactive material. When working with radioisotopes we are often concerned with the amount of radioactive decay that is occurring rather than the mass of the element that is present. Remember an element can have several isotopes, only some of which are unstable and decay. The unit system for radioactivity tells us the expected amount of radioactive emissions per unit of time. One Curie (Ci) corresponds to \(3.7 \times 10^{10}\) disintegrations per second.] After typing in “Curies” for units. The "?" inside the SOURCE stock icon in the equation view should now have disappeared.

Double-click on FRACTION DECAYING. Another definition box should open up. The upper left box now lists Required Inputs. Converters are a bit different from stocks. You can only use variables connected to a converter to define it. Since there are no connections going to FRACTION DECAYING, none are required. For this exercise, the source isotope of interest is radium 226, a relatively common trace element in the earth’s crust. \(^{226}\text{Ra}\) has a half-life of 1620 years, corresponding to a decay rate of \(4.279 \times 10^{-4}/\text{yr}\). Simply type in the value of FRACTION DECAYING, 0.0004279. Remember to enter the units: \{/year\}. 


This completes the model. You should have no more question marks on your Equation view. Now let's see what we have created.

All of your equations are listed in the Equation view. Look at SOURCE. Since it is a stock, it has two equations: the initial equation and the OUTFLOWS equation which the computer setup for you. Look at the equation next to SOURCE:

\[
\text{SOURCE}(t) = \text{SOURCE}(t - dt) + (-\text{DECAY\_RATE}) \times dt.
\]

Each incremental time period, STELLA will take the current value of SOURCE and subtract from it the amount that has decayed. The amount that has decayed is equal to the time step (dt) multiplied by the DECAY\_RATE. This new value of SOURCE becomes the current value for the next time increment. In our model, the units for time will be in years.

Double check your equations here. If you spot any typos, you can correct an equation by double-clicking on the icon in question in the Model layer. This returns you to the definition box and you can change the equation. When you are satisfied with your equations, return to the Model layer.

Click on the icon for the Graph Pad and release. Click somewhere on your diagram; and an “Untitled Graph” will appear. Note that the x-axis has a maximum of 12.00 units and the y-axis has no units, these are default settings. We will need to tell STELLA what we want to plot and what time period the simulation will run for.

Double click on the graph. A definition box will appear with a list of allowable things you can plot and a list of those that have been selected for this graph (presently, none are selected). We want to plot the amount of SOURCE over time, so click on SOURCE under the list of allowable items. Then, click on the >> symbol to transfer SOURCE to the Selected category.
Click on SOURCE in the Selected box to highlight it, then click on the up/down arrow (↑) to the right of SOURCE. The Scale box will be highlighted. To indicate the range, type in 0 for the minimum (or click to the right of 0.00 if this value is already shown as the default), hit TAB on the keyboard, then type 1000 for the maximum, if it is not already shown. Click on SET to save these values. Click on OK. The Graph pad will now appear on the screen.

The graph is named "Untitled" because you didn't name it. Double click on the graph pad to make the definition box reappear. Click on the graph title rectangle in the definition box, drag the mouse over the default name (Untitled) to select it, and type in a name of your choice ("Radium decay"?). Click on OK to return to the graph. **Note that several different variables can be plotted on one graph.** You can create additional graphs, if you
choose, by using the “Page” command in the graph definition box and giving them different names. And, you can also create different graphs using the Graph icon more than once in the Model window.

Now, let’s run our model. Click on the symbol that looks like a lollipop near the top left corner of your graph. This will deposit, or “pin,” the graph on your model view so that you can watch it as the model runs. Now click on Run under the Run menu, and release the click. You can watch on the Graph Pad as decay reduces the amount of the source. If you are fast, you can use the Run menu to Pause or Stop the simulation while it is in progress.

For the twelve year simulation period it looks as if the source does not go away. Let’s change the simulation time to 7000 years. Go to the Run menu. Click then select "Run Specs...", and click again. You will see another definition box. Click-and-hold at the right edge of the To value. Drag the mouse to the left to highlight this time. Type in 7000. The time step (DT) value allows you to vary the accuracy of your solution. Smaller values of DT are more accurate, but it will take longer for your model to run. Set DT to 1.0. Select time units of Years. Click on OK.

Try running your model again with this new setting. Use Run in the Run menu. Each time you run a model, it writes over the previous graphs and tables, so if you need to remember any information, make printouts of the graphs and/or tables before changing the model parameters. With the 7000 year simulation interval, you can see that the decay of the source as it slowly goes away. To speed up your simulation, return to the run menu and stop your run if it is still going. Then, click on Run Specs. Look at the simulation speed (Sim Speed). Initially, the model is using 0.1 seconds real time for each 1 year increment. The model will run faster if you instead use 0.01 seconds for each year, and still faster if you use 0.001 seconds per year. Change the speed, and run your model again; it should be faster.
You can get numerical data values for the amount of SOURCE remaining over time from the Table Pad. Return to the Model Construction layer. Click on the Table Pad icon and deposit it by clicking on an open area of the Model layer. An “Untitled Table” will appear.

Double click on the Table to define it. Choose SOURCE by clicking on it; then click on >> to transfer it to the Selected category. We want our table to have a vertical orientation; click on the vertical box if it does not have a ✓ in it. Click on OK. Move the table so that it is positioned somewhere on your model view, and click on the symbol that looks like a lollipop to “pin” the table on your model view.

Try running your model. After watching a few numbers fly by in the table, stop the run using the Run menu. As you may have noticed, STELLA has broken up the table by the DT time increment (i.e., 1 year). We do not need to see every year in the table; once every 500 years is sufficient.

To change the interval setting, return to the Table Pad definition box by double clicking on the table. Note that the “Report Interval” has a ✓ in the box that says "Every DT". Remove the ✓ by clicking on it. Type in 500 in the Report Interval box that appears after you remove the ✓. Click on OK. Return to the Table Pad and run your model again. Source should now be listed at 500 year intervals.

Please use your model to answer the following questions:
1) With your original amount of 900 Curies of $^{226}\text{Ra}$ and a decay rate of 0.0004279/year, how much source will remain in 1000 years, 2000 years, and 10,000 years? Note, you will need to change the simulation time to 10,000 years to answer the last part of this question. At the end of this assignment, I summarize what you need to turn in. In addition to the answer to the above question you will need to turn in a print out of your model diagram for $^{226}\text{Ra}$, your listing of equations, as well as a graph and a table for your simulation.
IMPORTANT TIP! Printing your results using the STELLA program at a CIT facility can result in generating a lot of pages (and the ensuing page charges, plus the guilt associated with killing all those trees). The STELLA software will require you to use a fresh page for each object or layer. One procedure for reducing the number of printed pages is to copy requested diagrams, equations, graphs and tables (as summarized below) and paste them into a word processing program (such as MS Word). This will allow you to resize each item and to put more than one item on a page. Graphs can be copied when they are unpinned. Data in tables must be selected before they can be copied. Select all of the model diagram before copying it by using the mouse to click above and to the left of the diagram and then dragging to the right and down before releasing your click. The text of your equations can be copied if selected (but the cute little icons will not be copied). Another approach is to carefully place your model, graph and table on the Model layer in STELLA and print that layer. This will give you those three items on one page (as long as they fit into the available space). Equations will have to be printed separately Alternatively my favorite, take a screen shot of your graph, table, or equations, then use an accessory such as Paint and cut and paste the portion of the screen image you want into your word processing program.

2) Now, let's change a couple of the parameters in your model. Go to the Model view. Double-click on the icon for SOURCE. A definition box should open up. Position the cursor just behind the last zero in 900. Click-and-hold. While still holding down the mouse button, move the mouse to the left so that all of 900 becomes highlighted. Release the click. Type in 400. Make sure your units are still in brackets. Now, the initial amount of source is different. Leave the definition box and return to the Model view by clicking on the arrow at the middle of its left edge. Double-click on FRACTION_DECAYING. Use the mouse to position the cursor, just behind the 9 in 0.0004279. Click-and-hold. Move the mouse to the left to highlight all of 0.0004279. Type in 66.143.

Now, you have adjusted your model to simulate a smaller amount of a radioactive isotope (400 Curies) that decays faster. The decay rate you have just typed in corresponds to that for Radon, the radioactive gas that is of concern to humans ($^{222}$Rn has a decay rate of 66.143/yr, i.e., a 3.825 day half life). Given the short half life of Radon it would be desirable to shorten the simulation time. Redefine the simulation period to be for a ½ year period. Incremental time steps (DT) of 1 year are now no longer appropriate; redefine the differential time increment to be 0.001 year. The Graph can be changed so that the maximum value for SOURCE is 500. The Table will also need to be changed so that the time increments are smaller; try using a report interval of 0.1 year. Run your model again. Go to the Table Pad and check the results. Note that the default format for numbers in the Table pad is two decimal places. You may find that Radon gets so small that the amount shows up as 0.00. To see the actual amount of
Radon in the table you will need to change the format for the SOURCE column. Double click on the SOURCE column and a format box will appear. Select **Free Float** for the numeric format under “precision”. Scientific notation will now be used to display the smaller numerical values. If you have trouble seeing the complete numerical value for the waste in the table, you can click on the right margin of the SOURCE column and drag it to the right to increase the column width. Print out the new graph and table.

**What fraction of the initial amount of Radon will be left after 0.1 year?**

---

**What did STELLA do?**

So far, in the above exercise you have used STELLA to solve a simple differential equation. Essentially your model for the SOURCE was as follows: \( \frac{dS}{dt} = -kS \)

where \( \frac{dS}{dt} \) is the instantaneous rate of change of the SOURCE

and \( k \) = the decay rate constant.

Your boundary conditions for solving this equation are that there is some initial amount of SOURCE (\( S_0 \)) at time = 0 and some other amount (\( S \)) at time = t. Integration of the above equation with these conditions gives: \( S = S_0 e^{-kt} \). You can use this analytical solution to the decay equation to check the accuracy of the numerical solution calculated by STELLA. Remember use of smaller time steps (DT) in the numerical model will result in a closer approximation to the exact solution that is given by the above equation.

3) Here’s a more challenging problem. When some radioactive isotopes decay, the decay product is also radioactive. \(^{226}\text{Ra}\) is a good example. When it decays, it produces a radioactive “daughter” \(^{222}\text{Rn}\). [That’s why we named it the “SOURCE”]. The Radon that is formed from the decay of Radium in soil can travel as a gas from the point of its generation. \(^{222}\text{Rn}\) can thus become a significant indoor air pollution problem in homes built over high-radium-containing minerals. Evidence indicates that inhalation of three of the decay products of \(^{222}\text{Rn}\) (\(^{218}\text{Po}, ^{214}\text{Pb}, \) and \(^{214}\text{Bi}\)) can result in an increased risk of lung cancer.

Return to the Model Construction layer and modify your diagram so that the model reflects both production and decay of the \(^{222}\text{Rn} daughter isotope product of \(^{226}\text{Ra}\). To do this you will need to add a new stock, plus more flows and converters. [If you need to remove some element from the former version of your model, you can do this by selecting the dynamite icon from the tools and by clicking it on the part of the model that you want to blow up.] Think about how to lay out the model diagram so that the
arrangement of stocks and flows reflect the sequential decay process. [Hint the decay of radium produces radon so the flow out of the source needs to flow into the inside of the stock for radon instead of out into the clouds.] Run the model for the case where the initial Ra is 900 Curies and the initial amount of Rn is zero. Use a 10 year simulation period (and remember all decay rates need to be in compatible time units). Use a time step (DT) of 0.001 years. Below you will be asked to turn in a printout of your revised model diagram, equations, and a table with values for Ra, and Rn at 1 year intervals. You will also need to create and turn in a graph of Ra, and Rn (note that you will need to define separate vertical scales for each isotope). While the maximum value for Ra is 1000 Curies, the suggested maximum for Rn is 0.01 Ci. Use zero as the minimum for both variables. Remember that the define graph dialog is where you can define the scale for each variable by clicking on the up/down arrow (▲) to its right. After you run your model, your graph scale for Radon may just have values of 0 showing. This is because the default scale has a precision of 0. Double click on the Radon variable at the top of your graph. A variable format box will appear. Select Free Float for the Radon scale, and click OK. You will now be able to see the values in the Radon scale displayed.

As you will see from your graph, the combination of a large Ra source with a slow decay rate and the rapid decay of Rn results in a virtually constant level of Rn gas in the soil.

The sequential decay problem you just solved involves three coupled ordinary differential equations (shown below). While an analytical solution exists for this particular case, it is cumbersome. The utility of STELLA is that it lets you generate a numerical solution to the problem with relative ease. Problems in the environmental field often involve coupled equations, where the rate of change of one variable depends on another that may also be changing, etc. STELLA can be a useful tool for models that describe these cases. We hope you will come to appreciate its ability to let you visualize and solve the problems we will be considering in CEE 3510.

**Coupled Differential Equations**

\[
\frac{dA}{dt} = -k_1 A \\
\frac{dB}{dt} = k_1 A - k_2 B \\
\frac{dC}{dt} = k_2 B - k_3 C
\]

Where \( A = ^{226}\text{Ra}; B = ^{222}\text{Rn}, C = ^{218}\text{Po}, ^{214}\text{pb}, \) and \(^{214}\text{Bi}\) and \(k_1, k_2, \) and \(k_3\) are the respective constants for fraction decaying (i.e., the rate constants for radioactive decay).

**Congratulations!** You have finished the first exercise. Use **Save As** (under the File menu) to keep the most recent version of your model. Give the version of the model
with sequential decay a different name. The “save as” command is suggested so that you can preserve the former version of your STELLA model in case you want to return to it later and make changes. If you are at a campus computing facility, be sure you saved the work on your own portable memory device. If you inadvertently saved onto the Hard Drive, delete that file (get the operator if you need help with this).

Please turn in the answers to questions in parts 1 and 2 (see table below). Also, turn in all STELLA printouts requested. Make sure your printouts include your list of equations, your model diagrams, as well as any graphs or tables that are requested.

Be sure to clearly indicate your solutions to the questions (i.e., don’t just hand in a print out of a table and expect the grader to read your mind to figure out what number(s) you think are important). On the transmittal sheet for your assignment, please indicate where you got your STELLA program, and which campus computing facility you used (if any). If you had any trouble getting or using the program, please tell us.

<table>
<thead>
<tr>
<th>Summary of STELLA output and information you should turn in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASE</td>
</tr>
<tr>
<td>900 Ci of $^{226}$Ra</td>
</tr>
<tr>
<td>400 Ci of $^{222}$Rn</td>
</tr>
<tr>
<td>900 Ci of $^{226}$Ra with $^{222}$Rn.</td>
</tr>
</tbody>
</table>