\[ P \cdot V = n \cdot R \cdot T \]

\[ P = \gamma h \]
\( \gamma \) is specific weight of water

\( V_{\text{tube}} = \text{volume of tube} \)

Find moles of air in tube at two different pressures. Difference must be supplied by pumping in air at atmospheric pressure.

Solve ideal gas law for \( n \) and substitute the equation for hydrostatic pressure to obtain the air pressure in the tube given submergence \( h \).

\[
n_1 = \left( \gamma h_1 + P_{\text{atm}} \right) \frac{V_{\text{tube}}}{R \cdot T}
\]

\[
n_2 = \left( \gamma h_2 + P_{\text{atm}} \right) \frac{V_{\text{tube}}}{R \cdot T}
\]

\[
n_2 - n_1 = \left( h_2 - h_1 \right) \frac{\gamma V_{\text{tube}}}{R \cdot T}
\]

The volume of air that must be supplied is

\[
V_{\text{pumped}} = \left( n_2 - n_1 \right) \frac{R}{P_{\text{atm}}} \frac{T}{P_{\text{atm}}}
\]

\[
V_{\text{pumped}} = \left( h_2 - h_1 \right) \frac{\gamma V_{\text{tube}}}{P_{\text{atm}}}
\]
Find pivot location $h_{\text{hinge}}$

First find location where pressure in the fluid is the same as atmospheric pressure. $h$ is distance above gage where pressure is atmospheric.

$$P_{\text{gage}} = \gamma h$$

$kPa := 1000Pa$  \hspace{1cm} $P = 1 \text{kPa}$

$$\gamma := 0.89806 \frac{N}{m^3}$$

$$h := \frac{P_{\text{gage}}}{\gamma}$$

$$h = 0.127 \text{m}$$

$$a := \frac{c}{\cos(\theta)}$$

$$a = 1.155 \text{m}$$

$$a$$ is height of gate along slant

$$b := 2 \text{m}$$

$b$ is width of gate

$$I_{xc} := \frac{b \cdot a^3}{12}$$

$$I_{xc} = 0.257 \text{m}^4$$

$$A_{\text{gate}} := a \cdot b$$

$$A_{\text{gate}} = 2.309 \text{m}^2$$

$$y_c := \frac{h + \frac{c}{2}}{\cos(\theta)}$$

$$y_c = 0.725 \text{m}$$

$$y_R = \frac{I_{xc}}{y_c \cdot A_{\text{gate}}} + y_c$$

$$y_R := \left[ \frac{b \cdot a \cdot \cos(\theta)}{12 \left( h + \frac{c}{2} \right) a \cdot b} + \frac{h + \frac{c}{2}}{\cos(\theta)} \right]$$

$$y_R = 0.878 \text{m}$$

$$h_{\text{hinge}} := h + c - y_R \cdot \cos(\theta)$$

$$h_{\text{hinge}} = 0.367 \text{m}$$
Find the flow rate \( Q \) from the opening in the tank.

Use Bernoulli’s equation. Place one point on the air-water interface inside the tank and the other point at the center of the port. Define the coordinate system with \( z=0 \) at the center of the port. Use atmospheric pressure as the pressure datum.

\[
\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g}
\]

\[
\frac{P_1}{\gamma} + z_1 = \frac{V_2^2}{2g}
\]

\[
V_2 := \left( \frac{2gP_1}{\gamma} + 2g z_1 \right)^{0.5}
\]

\[
V_2 = 8.355 \frac{m}{s}
\]

\[
Q := V_2 \frac{\pi d^2}{4}
\]

\[
Q = 2.625 \times 10^{-3} \frac{m^3}{s}
\]

\[
Q = 2.625 \frac{L}{s}
\]