**Nutrient Removal Project**

The nutrient removal project is an opportunity to synthesize what you have learned about environmental engineering and to learn about process control, real time data analysis, and the design and operation of a simple wastewater treatment plant.

This project will span several weeks during which time a team of students will select a treatment process, approximate the process using a sequencing batch reactor, develop the control algorithms for automation, operate the plant for several weeks, and monitor the plant performance.

**Project Milestones**

- Measure oxygen transfer efficiency with clean water (Gas Transfer Experiment).
- Select a reactor type (10 cm diameter cylinder or rectangular tank).
- Select a reactor process that can be carried out in a single sequencing batch reactor (SBR).
- Convert the process from a continuous flow reactor into a SBR so it can be modeled in a single reactor.
- Write and test the control logic to automate the SBR (fill, aerate, settle, waste, etc.).
- Test the operation of the SBR using tap water.
- Begin treating the synthetic waste. Use activated sludge to get the treatment process jump-started.
- Frequently measure the volatile suspended solids using the gravimetric technique to verify that the reactor system is maintaining a healthy community of microbes.

**Research Options**

1) Develop an algorithm to control the dissolved oxygen concentration in the SBR. Compare the ability of several different control algorithms. Log the relevant parameters to a file for analysis.

2) Research algorithms used to control a process (see Dissolved Oxygen Control on page 125). Develop an algorithm to automate the measurement of the oxygen uptake rate. Measure the oxygen uptake rate periodically and generate a graph containing the uptake rate as a function of time. Log the measured oxygen uptake rate to file.

3) Research biological phosphorus removal techniques. Operate the plant to optimize phosphorus removal and measure phosphorus removal (See Phosphorus on page 131). Note that phosphorus measurements are done using a colorimetric wet chemistry technique that we can’t easily automate.

4) Develop a control strategy to maintain a desired level of volatile suspended solids in the SBR. Consider using the Honeywell turbidity sensors or the Hach turbidimeter to automatically measure the turbidity and use the turbidity measurement to estimate the volatile suspended solids.
Reactor Specifications
Each team will use a single reactor for the wastewater treatment plant. The maximum reactor volume will be 4 L. Aeration will be provided using a solenoid valve and accumulator to meter air from the laboratory low-pressure air supply. A magnetic stirrer will mix the reactor contents and will be controlled (on and off) by the NRP software.

Reactor Designs for Denitrification
The following reactor designs are from (Rittmann and McCarty 2001).

Tertiary Denitrification using Activated Sludge
Solids Retention Time, SRT, of 5 d
High cell concentration increases reaction rate

![Diagram of Tertiary Denitrification using Activated Sludge](image)

Electron donor

$\text{NO}_3^-$

$N_2$

No aeration!

Solids recycle

Biofilm Processes
Submerged fixed beds of rocks, sand, limestone, or plastic media
Fluidized beds of sand, activated carbon, and pellets of ion-exchange resin
Circulating beds of a range of lightweight particles
Membrane bioreactors (membrane supplies H$_2$ and is the attachment surface)
HRT can be less than 10 minutes!

Biomass storage and decay
Uses biomass as electron donor for denitrification
Slow kinetics of endogenous decay

![Diagram of Biofilm Processes](image)

$\text{BOD}^0$

$\text{TKN}^0$

$CO_2$

$\text{Biomass}$

$\text{NO}_3^-$

$N_2$

Low $\text{BOD}^0$

Low $\text{NH}_4^+$

Some $\text{NO}_3^-$

Sludge recycle

Sludge waste

Nutrient Removal Project
**Classical pre-denitrification**
Uses BOD as electron donor for denitrification
Requires high mixed liquor recycle (4Qplant)

**Simultaneous nitrification with denitrification**
Uses BOD as electron donor
Low oxygen levels permit denitrification

**Barnard Process**
Greater than 90% removal of TKN!

**Sequencing Batch Reactor**
Same process as Barnard carried out in a single tank
Feed Composition

The feed composition is based on a synthetic feed used by (Cicek, Franco et al. 1998). The feed was divided into 3 stock solutions to simplify preparation. The inorganic stocks (Stocks 2 and 3) will be added to tap water in a 100 L tank that will be pumped to each plant. Stock 1, containing the biodegradable organics, will need to be refrigerated. Stock 1 will be diluted to a 20x stock and will be stored in a small container in a refrigerator located near the sequencing reactor. The diluted organic stock will be metered into the plant by gravity. The amount of stock metered into the plant will be controlled by either measuring the pressure at the bottom of the stock bottle or by measuring the increase in pressure in the sequencing batch reactor. The advantages of using a slightly more dilute organic stock are that more of the starch is soluble and that it is easier to measure the volume of the organic stock that must be added to the sequencing batch reactor.

The compositions of the 3 stock solutions are given in Tables 10-1 to 10-3.

Table 10-1. Composition of synthetic feed stock 1.

<table>
<thead>
<tr>
<th>Compound</th>
<th>Chemical Formula</th>
<th>Molecular Weight g/mol</th>
<th>Concentration mg/L</th>
<th>Stock Concentration (100x) g/L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starch</td>
<td></td>
<td>~40,000</td>
<td>84.40</td>
<td>8.440</td>
</tr>
<tr>
<td>Casein</td>
<td></td>
<td>~30,000</td>
<td>125.00</td>
<td>12.500</td>
</tr>
<tr>
<td>Sodium acetate</td>
<td>C₂H₅O₂Na·3H₂O</td>
<td>136.1</td>
<td>31.90</td>
<td>3.190</td>
</tr>
<tr>
<td>Capric acid</td>
<td>C₁₀H₂₀O₂</td>
<td>172.3</td>
<td>11.60</td>
<td>1.160</td>
</tr>
<tr>
<td>Ammonium chloride</td>
<td>NH₄Cl</td>
<td>53.5</td>
<td>75.33</td>
<td>7.533</td>
</tr>
<tr>
<td>Potassium phosphate</td>
<td>K₂HPO₄</td>
<td>174.2</td>
<td>6.90</td>
<td>0.690</td>
</tr>
<tr>
<td>Sodium hydroxide</td>
<td>NaOH</td>
<td>40.0</td>
<td>175.00</td>
<td>17.500</td>
</tr>
<tr>
<td>Glycerol</td>
<td>C₃H₈O₃</td>
<td>92.1</td>
<td>12.00</td>
<td>1.200</td>
</tr>
</tbody>
</table>
Table 10-2. Composition of synthetic feed stock 2.

<table>
<thead>
<tr>
<th>Compound</th>
<th>Chemical Formula</th>
<th>Molecular Weight g/mol</th>
<th>Concentration mg/L</th>
<th>Stock Concentration (1000x) g/L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnesium sulfate</td>
<td>MgSO₄·7H₂O</td>
<td>246.5</td>
<td>69.60</td>
<td>69.600</td>
</tr>
<tr>
<td>Sodium molybdate</td>
<td>NaMoO₄·2H₂O</td>
<td>241.9</td>
<td>0.15</td>
<td>0.150</td>
</tr>
<tr>
<td>Manganese sulfate</td>
<td>MnSO₄·H₂O</td>
<td>169.0</td>
<td>0.13</td>
<td>0.130</td>
</tr>
<tr>
<td>Cupric sulfate</td>
<td>CuSO₄·4H₂O</td>
<td>249.7</td>
<td>0.08</td>
<td>0.080</td>
</tr>
<tr>
<td>Zinc sulfate</td>
<td>ZnSO₄·7H₂O</td>
<td>287.5</td>
<td>0.48</td>
<td>0.480</td>
</tr>
</tbody>
</table>

Table 10-3. Composition of synthetic feed stock 3.

<table>
<thead>
<tr>
<th>Compound</th>
<th>Chemical Formula</th>
<th>Molecular Weight g/mol</th>
<th>Concentration mg/L</th>
<th>Stock Concentration (1000x) g/L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calcium chloride</td>
<td>CaCl₂·2H₂O</td>
<td>147.0</td>
<td>22.50</td>
<td>22.500</td>
</tr>
<tr>
<td>Iron chloride</td>
<td>FeCl₃·6H₂O</td>
<td>270.3</td>
<td>18.33</td>
<td>18.330</td>
</tr>
<tr>
<td>Cobalt chloride</td>
<td>CoCl₂·6H₂O</td>
<td>237.9</td>
<td>0.42</td>
<td>0.420</td>
</tr>
</tbody>
</table>

Completely soluble at feed concentration
325 mg/L COD (Chemical Oxygen Demand)
40.9 mg/L nitrogen
Stocks 2 and 3 will be added to tap water in the 100 L tank that will be pumped to each plant
Stock 1 will be diluted to a 20x stock and refrigerated and metered into your plant by gravity or by a computer controlled peristaltic pump.

Measuring Suspended Solids

Suspended solids measurements are an easy way to track plant performance. The minimum national standard for secondary wastewater treatment is that the average 30-day concentration of total suspended solids be less than 30 mg/L. The clarified plant effluent can be monitored to determine the ability of the sequencing batch reactor to comply with this standard. In addition, a well-operated plant will need a high level of volatile suspended solids (VSS) in the sequencing batch reactor to achieve rapid biochemical oxidation of the organic matter. A suggested level of VSS in the sequencing batch reactor is between 2 and 3 g/L.

Two techniques can be used to measure total suspended solids. The most direct technique is by filtered a sample and then weighing the dried solids.
Total Suspended Solids (TSS) Protocol

1) Using filter tweezers, transfer a prewashed and dried filter (Whatman GF/or equivalent) from a desiccator to balance and determine tare weight to nearest 0.1 mg.

2) Place in a Millipore® filtration apparatus, or equivalent, and apply suction.

3) Pour through 50-mL sample (or other suitable volume).

4) Wash sample container, then the filter holder and filter, with two 10-mL portions of distilled water.

5) Carefully remove filter with tweezers.

6) Dry 1 hr at 103-105°C (use an aluminum dish to hold filter in drying oven and to provide identification of filter).

7) Cool in desiccator and reweigh to nearest 0.1 mg.

8) Compute mg/L TSS as (mass of residue)/(volume of initial sample).

Turbidity Measurement of TSS

Turbidity measurements are an indirect technique that can be automated and correlated with the gravimetric technique. Although there is not a general relationship between turbidity and TSS an approximate empirical relationship can be developed for a particular combination of wastewater and turbidity sensor. The relationship can be assumed to be linear.

\[
TSS \left[ \frac{mg}{L} \right] = C \left[ \frac{mg}{NTU} \right] \cdot Turbidity \left[ NTU \right]
\]  

where \( C \) has an approximate value of 2.3 \( \left[ \frac{mg}{L} \right] \) for settled secondary effluent (Metcalf & Eddy, Tchobanoglous et al. 2003).

The laboratory inline turbidity sensors from Honeywell could be used to measure the reactor effluent turbidity. For accurate measurements it is essential that the sensor be plumbed so that air cannot accumulate in the sensor. It is possible that the sensor will need to be cleaned occasionally to remove biofilm growth on the sensor optics.

Process Control

An introduction to automated process control is available in pages 1681 to 1703 of (Metcalf & Eddy, Tchobanoglous et al. 2003).

Sensors

Sensors that are can be used to monitor the status of the sequencing batch reactor include pressure, temperature, dissolved oxygen, pH, and turbidity. The pressure sensors are quite versatile and can be used to measure airflow, water flow, volume of water in the reactor, as well as head loss through the course bubble diffuser.
States

Process control software uses a programming structure called a “State Machine” to control a sequence of events. The NRP software uses a state machine in the Plant Control VI. The Plant Control VI is called inside a while loop. The while loop iterates approximately once every 50 ms. The Plant Control VI remembers what state it is currently in using a shift register in a while loop that executes once each time Plant Control VI is called. More information on state machines is available at.

Airflow Control

The NRP software controls the flow of air into the sequencing batch reactor. The control algorithm is based on the assumption that the relationship between pressure in the flow accumulator and flow through the porous stone should be relatively constant.

Calibration

A calibration routine evaluates the relationship between pressure and flow. The NRP software acquires data continuously and stores the most recent 5000 data points in a buffer. The data collection process continues in the normal fashion during the calibration routine. The inlet air valve is opened until the pressure reaches the maximum value and then the system waits until the pressure drops to the minimum value. When the pressure reaches the minimum valve a calibration analysis SubVI is called.

The calibration SubVI retrieves all of the data from the data buffer. Searching for the maximum pressure starting from the most recent data identifies the data that corresponds to the calibration. The data up to the maximum pressure is divided into a number of pressure ranges according to the value of “line segments.” A linear fit is created for each pressure range and the slopes (Δp/Δt) are stored in one array and the mean pressures for the range are stored in a 2nd array. A polynomial fit is created for Δp/Δt as a function of pressure. The polynomial fit may be first order, a straight line, if the majority of the losses are due to laminar flow losses. If the flow regime is turbulent a 2nd order polynomial may work well. If the flow regime covers both laminar and turbulent regimes a higher order polynomial will be required.

The polynomial fit is used to estimate the rate of pressure drop as a function of the measured pressure in the accumulator. The flow rate out of the accumulator is calculated using the ideal gas law.

\[
\frac{\Delta P}{\Delta t} V = \frac{\Delta n}{\Delta t} RT
\]

Rearranging equation 10.2 and solving for the flow rate given by the change in moles/unit time

\[
\frac{\Delta n}{\Delta t} = \frac{\Delta P}{\Delta t} \frac{V}{RT}
\]
where \( \frac{\Delta P}{\Delta t} \) is a function of pressure in the accumulator, \( V \) is the accumulator volume, \( R \) is the ideal gas constant, and \( T \) is absolute temperature.

The relationship given by equation 10.3 is valid whether the accumulator is filling or emptying since the rate of pressure drop due to air flowing into the sequencing batch reactor is assumed to be only a function of the pressure in the accumulator.

**Valve Control Algorithm**

The NRP software controls the valve that fills the accumulator. The control algorithm is designed to ensure that the cumulative error (in moles of air) is minimized. The error term is calculated by taking the difference between the measured and target flow rates, multiplying by the \( \Delta t \) since the previous data, and maintaining a running sum of these errors. The error term is reset whenever a “zero error”, “write polynomial”, “configure”, or “uncalibrate” command is sent to the air flow integrator. The error term is also set to zero whenever the valve that controls the airflow is closed (such as in the settle state.) The target flow can be changed programmatically to achieve a dissolved oxygen level by sending a “configure” command to the air flow integrator along with the flow control data.

The valve is opened by the flow control SubVI whenever the instantaneous flow rate is less than the target flow rate and the error is less than zero. The valve is closed when the instantaneous flow rate reaches the 105% of the target flow rate. The cycle repeats continuously as the flow accumulator is charged and discharged.

**Air Flow Fluctuation Minimization**

The flow fluctuations can be minimized for a given range of pressure fluctuations if the pressure fluctuations are small relative to the absolute pressure. Thus the best flow control will occur when the pressure in the flow accumulator is high. Increasing the flow resistance for the air leaving the accumulator can increase the air pressure in the accumulator that is required to deliver the desired flow. A manual needle valve (\( N_2 \) in Figure 10-2) can be used to adjust the accumulator pressure. The house air supply produces approximately 100 kPa and thus the maximum flow rate required should not require a pressure greater than approximately 80 kPa.

The fluctuation in airflow rate is also a function of the total latency between the pressure in the accumulator reaching the target value and the valve changing state. The largest latency is a potential 50 ms lag for communication with the valve through the Stamp microprocessor. The effect of this latency can be reduced by decreasing the airflow rate into the accumulator by increasing head loss (with a needle valve) in the...
line between the air supply and the flow control valve. The head loss could be set so the duty cycle (amount of time the valve is open) is approximately 50% for the maximum anticipated airflow rate.

**Air Flow Control Calibration Procedure**

1) Set up the airflow control hardware (Figure 10-2). The pressure sensor should be able to measure at least 100 kPa.

2) Configure the NRP software so that pressure sensor is calibrated to have an output in Pascals.

3) Configure the NRP software so the port that solenoid valve (S₁) is connected to is entered in the flow control data structure.

4) Set the “Maximum Pressure” to 80,000 Pa and the Minimum Pressure to 60,000 Pa (see Figure 10-1).

5) Turn on solenoid valves S₁ and S₂ in the aeration state in the plant method.

6) Set the operator selected state to aeration.

7) The solenoid valves should open and the pressure in accumulator should increase to the maximum pressure and then decrease to the minimum pressure. When the pressure reaches the minimum pressure a calibration SubVI dialog will open.

8) The calibration SubVI dialog contains a graph of pressure vs. time and a graph of Δp/Δt vs. time. Adjust the number of line segments and the order of the polynomial fit to get a good agreement between the polynomial and the Δp/Δt vs. time plot.

9) Observe the maximum flow rate (that corresponds to the maximum pressure used for calibration). If the maximum flow rate is less than the maximum flow rate that you need then decrease the flow resistance by opening needle valve N₂. If the maximum flow rate is greater than the maximum flow rate that you need then close needle valve N₂.

10) Click on the Calibrate Air Flow button to recalibrate the flow control with the new needle valve setting.

11) Repeat steps 7 to 10 until the maximum flow rate is close to the desired maximum flow rate.
12) Change the minimum air pressure from 60000 Pa to 10,000 Pa and repeat steps 7-9.

13) Increase the ability of the control software to close the valve before the pressure in the accumulator gets too large by slowing the fill rate of the accumulator. The fill rate can be decreased by slowly closing the needle valve N1 (Figure 10-2). Increase the fill time such that the control valve spends about half of the time open when it is maintaining the maximum desired flow rate.

14) Note that it is necessary to recalibrate any time the flow resistance changes on the effluent of the accumulator.

15) Note that it isn’t necessary to recalibrate when changing the flow resistance on the influent of the accumulator.

16) Note that a high order polynomial may be required to get a good fit because the relationship between pressure and flow rate covers the transition region from laminar to turbulent flow.

**Dissolved Oxygen Control**

The dissolved oxygen concentration is normally a controlled parameter in wastewater treatment plants. It can either be manually controlled by a plant operator or automatically controlled by a computer. Regulating the airflow rate controls the oxygen concentration. It is important to control the airflow because approximately 55% of the energy usage in a typical wastewater treatment plant employing the activated-sludge process is aerating the activated sludge tank (Metcalf & Eddy, Tchobanoglous et al. 2003)! Since the oxygen transfer efficiency decreases as the oxygen deficit decreases the highest efficiency can be obtained by keeping the dissolved oxygen concentration as low as possible.

The control algorithm could be based on feedback (sensor data) from the dissolved oxygen sensor and could change the target airflow (see Figure 10-1). The software will seek a new target airflow whenever the new flow control settings are sent to the airflow integrator using a “configure” command.

The oxygen concentration will change slowly in the SBR and thus the control loop for varying the airflow rate could operate at most once per minute. The major challenge is to develop an algorithm that modifies the airflow rate correctly when the measured dissolved oxygen concentration is not equal to the set point. Potential control strategies include on/off control, some combination of proportional, integral, and derivative control, or some other form of self-correcting control.

**On Off Control**

The simplest approach would be to turn the aeration system on and off based on whether the dissolved oxygen concentration is below or above the set point. This would be an inefficient strategy since the flow rate setting for “on” would have to be sufficient to aerate the SBR under the worst-case scenario. The oxygen transfer efficiency decreases as the flow rate increases and thus on/off control is inefficient. On/off control may also result in the significant oscillations of the dissolved oxygen concentration.
Proportional, Integral, and Derivate Control

Proportional-Integral-Derivative (PID) control sets the value of the control parameter (airflow in this case) based on the sum of the error, the integral of the error and the derivate of the error.

\[ u(t) = K_c \left( \varepsilon + \frac{1}{T_i} \sum \varepsilon \cdot \Delta t + T_d \frac{\Delta \varepsilon}{\Delta t} \right) \]

10.4

\[ u(t) \] is the airflow rate that the controller sets

The tuning parameters need to be adjusted simultaneously to tune the control loop. More detailed explanations of PID control are available at [http://www.ni.com/pdf/manuals/320563b.pdf](http://www.ni.com/pdf/manuals/320563b.pdf). PID VIs are available that would require very little modification for use with the NRP software.

Simulation Model Based Control

The SBR is simple enough that it should be possible to model the oxygen processes of consumption and reoxygenation. If we model the process analytically then the model can guide the selection of a new airflow rate based on recent system performance.

We’ll begin by developing the oxygen deficit as a function of time in a completely mixed batch reactor (no inflow and no outflow) with initial concentrations of Biochemical Oxygen Demand (BOD\(_I\)) and dissolved oxygen. We will include oxidation of BOD\(_I\) and reaeration. These effects are coupled in equation 10.5 where \( C \) represents oxygen concentration. The first two terms on the right are negative since oxidation of BOD and respiration consume oxygen while the third term is usually positive since reaeration increases the concentration of oxygen (except in the rare instance where the dissolved oxygen concentration is greater than the equilibrium dissolved oxygen concentration). Eventually we will make a comparison between time in a reactor and distance down a river.

\[ \frac{\partial C}{\partial t} = \frac{\partial C_{\text{oxidation}}}{\partial t} + \frac{\partial C_{\text{respiration}}}{\partial t} + \frac{\partial C_{\text{reaeration}}}{\partial t} \]

10.5

Oxidation of BOD

We must first develop a relationship for the change in oxygen concentration due to oxidation of organics. The rate that oxygen is used will be proportional to the rate that
substrate (or biochemical oxygen demand) is oxidized. The rate of substrate utilization by bacteria is given by the Monod relationship

\[
\frac{dL}{dt} = \frac{-kLX}{K_s + L} \tag{10.6}
\]

where \( L \) is substrate concentration expressed as oxygen demand or BOD \(_L\) [mg/L], \( k \) is the maximum specific substrate utilization rate, \( K_s \) is the half velocity constant, and \( X \) is the concentration of bacteria. If the half velocity concentration is large relative to the concentration of substrate we obtain

\[
\frac{dL}{dt} = \frac{-kXL}{K_s + L} \approx \left[ \frac{-kX}{K_s} \right] L \approx -k_{ox} L \tag{10.7}
\]

where \( k_{ox} \) is a first order oxidation rate constant that includes both the approximation that the bacteria concentration is roughly constant and that the substrate concentration is smaller than the half velocity constant.

Separate variables and integrate

\[
Lint \frac{dL}{L} = \int_{L_o}^{L} (-k_{ox})dt \tag{10.8}
\]

to obtain

\[
L = L_o e^{-k_{ox}t} \tag{10.9}
\]

The rate of oxygen utilization is equal to the rate of substrate utilization (when measured as oxygen demand) and thus we have

\[
\frac{\partial C_{oxidation}}{\partial t} = \frac{dL}{dt} = -k_{ox} L \tag{10.10}
\]

where \( C \) is the dissolved oxygen concentration [mg/L]. Now we can substitute for \( L \) in equation 10.10 using equation 10.9 to obtain

\[
\frac{\partial C_{oxidation}}{\partial t} = -k_{ox} L_o e^{-k_{ox}t} \tag{10.11}
\]

**Respiration**

Bacteria utilize oxygen for respiration and for cell synthesis. When no substrate is present the bacteria cease synthesis, but must continue respiration. This continual use of oxygen is termed "endogenous respiration." Bacteria use stored reserves for endogenous respiration. We can model this oxygen demand as a constant that is added to the demand for oxygen caused by substrate utilization. As a first approximation, we can assume that this oxygen demand is proportional to the concentration of bacteria. In addition, we will assume that the population of bacteria is relatively constant throughout the experiment.
where \( b \) is the specific endogenous oxygen consumption rate and \( k_e \) is the endogenous oxygen consumption rate.

**Oxygen Transfer Coefficient**

The rate of oxygen transfer is directly proportional to the difference between the actual dissolved oxygen concentration and the equilibrium dissolved oxygen concentration.

\[
\frac{\partial C_{\text{respiration}}}{\partial t} = -bX = -k_e t
\]

\[
10.12
\]

where \( C^* \) is the equilibrium oxygen concentration, \( C \) is the actual dissolved oxygen concentration, and \( \hat{k}_{v,j} \) is the overall volumetric oxygen transfer coefficient. If reaeration is the only process affecting the oxygen concentration then equation 10.13 can be integrated to obtain

\[
\ln \frac{C^* - C}{C^* - C_0} = \hat{k}_{v,j}(t - t_0)
\]

\[
10.14
\]

**Oxygen Deficit**

We now have equations for the reaction of oxygen with \( \text{BOD}_l \), endogenous respiration, and for reaeration. Substituting into equation 10.5 we get

\[
\frac{\partial C}{\partial t} = -k_{ox} L_e e^{-k_{ox} t} k_e + \hat{k}_{v,j} (C^* - C)
\]

\[
10.15
\]

We can simplify the equation by defining oxygen deficit \( D \) as:

\[
D = C^* - C
\]

\[
10.16
\]

and noting that the rate of change of the deficit must be equal and opposite to the rate of change of oxygen concentration

\[
\frac{dC}{dt} = -\frac{dD}{dt}
\]

\[
10.17
\]

We must remember that the deficit can never be greater than the equilibrium concentration (\( D \) must always be less than \( C^* \))! In addition, the BOD model breaks down if the dissolved oxygen concentration is less than about 2 mg/L because the lack of oxygen will limit microbial kinetics and \( \frac{dL}{dt} \) will no longer equal \(-k_{ox} L_e \). If we stick to conditions under which our assumptions are valid then we can substitute equations 10.16 and 10.17 into equation 10.15 to obtain

\[
\frac{\partial D}{\partial t} = k_e + k_{ax} L_e e^{-k_{ax} t} - \hat{k}_{v,j} D
\]

\[
10.18
\]
Equation 10.18 describes the rate of change in the oxygen deficit as a function of the rate of biological consumption of oxygen and the rate of reoxygenation. The two terms describing biological consumption will be difficult to model separately and can be combined into a single term.

\[
k_{\text{consumption}} = k_e + k_{\text{air}} L_o e^{-k_{\text{air}} t}
\]  \hspace{1cm} 10.19

The rate of oxygen consumption varies with time during the wastewater treatment process. It can be calculated as a function of the measured oxygen concentration. Substitute equation 10.19 into equation 10.18, replace the partial derivative with a finite difference and solve for the unknown oxygen consumption rate.

\[
k_{\text{consumption}} = \left( \frac{\Delta D}{\Delta t} \right)_i + \hat{k}_{\text{air},i} (\dot{n}_{\text{air},i}) D_i
\]  \hspace{1cm} 10.20

The oxygen deficit can easily be measured as a function of time using a dissolved oxygen probe. The change in the deficit with time can be calculated using data acquired over some appropriate time interval. The oxygen transfer coefficient, \(\hat{k}_{\text{air}}(\dot{n}_{\text{air}})\), was measured as a function of the airflow rate in the Gas Transfer experiment.

A new target oxygen transfer coefficient can be calculated based on the goal of reaching the target oxygen deficit in some user defined time interval.

\[
\hat{k}_{\text{air},i} (\dot{n}_{\text{air},(i+1)}) = \frac{k_{\text{consumption}} - \left( \frac{\Delta D}{\Delta t} \right)_{(i+1)}}{D_{(i+1)}}
\]  \hspace{1cm} 10.21

where the change in deficit is based on the goal of reaching the target deficit, \(D_{(i+1)}\), by the end of the time interval \(\Delta t\).

\[
\left( \frac{\Delta D}{\Delta t} \right)_{(i+1)} = \frac{D_{(i+1)} - D_i}{\Delta t}
\]  \hspace{1cm} 10.22

Substituting equations 10.20 and 10.22 into equation 10.21 we can obtain a final equation that gives a new oxygen transfer coefficient based on the target deficit, the measured deficit, the measured rate of change in the deficit and the target rate of change in the deficit.

\[
\hat{k}_{\text{air},i} (\dot{n}_{\text{air},(i+1)}) = \frac{1}{D_{(i+1)}} \left[ \left( \frac{\Delta D}{\Delta t} \right)_i - \left( \frac{\Delta D}{\Delta t} \right)_{(i+1)} + \hat{k}_{\text{air},i} (\dot{n}_{\text{air},i}) D_i \right]
\]  \hspace{1cm} 10.23

The term \(\hat{k}_{\text{air},i} (\dot{n}_{\text{air},(i+1)})\) is the new required oxygen transferred coefficient. Since the oxygen transfer coefficient is a function of the airflow the new airflow can be determined.
One of the advantages of using equation 10.23 to control the airflow is that the oxygen consumption rate is calculated. The oxygen consumption rate could be used as an indicator of the extent to which the influent BOD has been oxidized.

**Algorithm to Measure the Oxygen Uptake Rate**

One of the objectives of a wastewater treatment plant is to reduce the Biochemical Oxygen Demand (BOD). The minimum national standard for secondary wastewater treatment is that the average 30-day concentration of BOD₅ be less than 30 mg/L. Biochemical oxygen demand is difficult to measure since it takes 5 days for a test. The long test period also precludes the possibility of using BOD as a control parameter in operating a WWTP. Most WWTPs don’t have the luxury of knowing the concentration of influent BOD. For the NRP the composition and properties of the synthetic feed are known. Thus it should be possible to estimate the BOD removal and the residual BOD by measuring the oxygen uptake rate. Temporarily increasing the oxygen concentration in the sequencing batch reactor, turning the airflow off, and then measuring the decrease in oxygen concentration with time can measure the oxygen uptake rate. The aeration rate with the airflow turned off is insignificant and thus the rate of oxygen consumption is equal to the rate of change of the oxygen concentration.

**References**

