13.53 Information and assumptions

A rectangular weir is being designed for $Q = 4 \text{ m}^3/\text{s}$, $L = 3 \text{ m}$. Water depth upstream of weir is 2 m.

**Find**

Weir height: $P$

**Solution**

First guess $H/P = 0.60$. Then

$$K = 0.40 + 0.05(0.60) = 0.43.$$

For a fully ventilated weir.

$$Q = K \sqrt{2gLH^{3/2}}$$

Solve for $H$:

$$H = \frac{Q}{(K \sqrt{2gL})^{2/3}}$$

$$= \frac{4}{(0.43 \sqrt{(2)(9.81)(3)})^{2/3}} = 0.788 \text{ m}$$

Iterate:

$$H/P = 0.788/(2 - 0.788) = 0.65; \quad K = 0.40 + .05(.65) = 0.433$$

$$H = \frac{4}{(0.433 \sqrt{(2)(9.81)(3)})^{2/3}} = 0.785 \text{ m}$$

Thus:

$$P = 2.0 - H = 2.00 - 0.785 = 1.215 \text{ m}$$
15.4 Information and Assumptions

provided in problem statement

Find

the Froude number, type of flow and critical depth

Solution

\[ Q = VA \]
\[ 12 \text{ m}^3/\text{s} = V(3 \times y) \]
\[ V_{0.30} = 12 \text{ m}^3/\text{s} / (3 \times 0.30 \text{ m}) = 13.33 \text{ m/s}; \]
\[ V_{1.0} = 12 \text{ m}^3/\text{s} / (3 \times 1 \text{ m}) = 4 \text{ m/s} \]
\[ V_{2.0} = 12 \text{ m}^3/\text{s} / (3 \times 2 \text{ m}) = 2 \text{ m/s} \]
\[ Fr_{0.3} = 13.33 \text{ m/s} / (9.81 \text{ m/s}^2 \times 0.30 \text{ m})^{1/2} = 7.77 \] (supercritical)
\[ Fr_{1.0} = 4 \text{ m/s} / (9.81 \text{ m/s}^2 \times 1.0 \text{ m})^{1/2} = 1.27 \] (supercritical)
\[ Fr_{2.0} = 2 \text{ m/s} / (9.81 \text{ m/s}^2 \times 1.0 \text{ m})^{1/2} = 0.452 \] (subcritical)

The critical depth is

\[ y_c = (q^2/g)^{1/3} = ((4 \text{ m}^2/\text{s})^2 / (9.81 \text{ m/s}^2))^{1/3} \]
\[ = 1.18 \text{ m} \]
15.16 Information and Assumptions

from Fig. 15.7 $C \approx 0.85$

provided in problem statement

Find

the water surface elevation

Solution

\[ Q = 0.385C \, L \sqrt{2gH^{3/2}} \]
\[ 25 = 0.385(0.85)(10)\sqrt{2 \times 9.81H^{3/2}} \]
\[ (H)^{3/2} = 1.725 \]
\[ H = 1.438, \]  \hspace{1cm} (10)

Water surface elevation = 101.4 m
15.18 Information and Assumptions

provided in problem statement

Find
change in depth and maximum size of upstep

Solution

\[ E_1 = y_1 + \frac{V_1^2}{2g} = 3 + \frac{3^2}{2 \times 9.81} = 3.46 \text{ m} \]
\[ F_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{3}{\sqrt{9.81 \times 3}} = 0.55 \text{ (subcritical)} \]

Then

\[ E_2 = E_1 - \Delta z_{step} = 3.46 - 0.30 = \underline{3.16} \text{ m} \]

\[ y_2 + \frac{q^2}{(2g y_2^2)} = 3.16 \text{ m} \]
\[ y_2 + \frac{9^2}{(2g y_2^2)} = 3.16 \]
\[ y_2 + 4.13/\overline{y_2^2} = 3.16 \]

Solving for \( y_2 \) yields \( y_2 = 2.49 \text{ m} \). Then

\[ \Delta y = y_2 - y_1 = 2.49 - 3.00 = -0.51 \text{ m} \]

So water surface drops \( 0.51 \text{ m} \).

For a downward step

\[ E_2 = E_1 + \Delta z_{step} = 3.46 + 0.3 = \underline{3.76} \text{ m} \]
\[ y_2 + 4.13/\overline{y_2^2} = 3.76 \]

Solving for \( y_2 \) gives \( y_2 = 3.40 \text{ m} \). Then

\[ \Delta y = y_2 - y_1 = 3.40 - 3 = \underline{0.40} \text{ m} \]

Water surface elevation change = \( +0.10 \text{ m} \)

Max. upward step before altering upstream conditions:

\[ y_c = y_2 = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{9^2}{9.81}} = 2.02 \]
\[ E_1 = \Delta z_{step} + E_2 \]

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where

\[ E_2 = 1.5y_c = 1.5 \times 2.02 = 3.03 \text{ m} \]

Maximum size of step

\[ z_{\text{step}} = E_1 - E_2 = 3.46 - 3.03 = 0.43 \text{ m} \]
15.21 Information and Assumptions

provided in problem statement

Find

change in depth and water surface elevation and greatest contraction allowable

Solution

\[ F_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{3}{\sqrt{9.81 \times 3}} = 0.55 \text{ (subcritical)} \]
\[ E_1 = E_2 = y_1 + \frac{V_1^2}{2g} = 3 + \frac{3^2}{2 \times 9.81} = 3.46 \text{ m} \]
\[ q_2 = \frac{Q}{B_2} = \frac{27}{2.6} = 10.4 \text{ m}^3/\text{s/m} \]

Then

\[ y_2 + \frac{q^2}{2gy_2^2} = y_2 + \frac{(10.4)^2}{2 \times 9.81 \times y_2^2} = 3.46 \]
\[ y_2 + 5.50/y_2^2 = 3.46 \]

Solving: \( y_2 = 2.71 \text{ m} \).

\[ \Delta z_{\text{water surface}} = \Delta y = y_2 - y_1 = 2.71 - 3.00 = 0.29 \text{ m} \]

Max. contraction without altering the upstream depth will occur with \( y_2 = y_c \)

\[ E_2 = 1.5y_c = 3.45; \ y_c = 2.31 \text{ m} \]

Then

\[ \frac{V_c^2}{2g} = \frac{y_c}{2} = 2.31/2 \text{ or } V_c = 4.76 \text{ m/s} \]
\[ Q_1 = Q_2 = 27 = B_2y_cV_c \]
\[ B_2 = \frac{27}{(2.31 \times 4.76)} = 2.46 \text{ m} \]

The width for max. contraction = 2.46 m
15.26 Information and Assumptions

provided in problem statement

Find

depth of water

Solution

\[ V = \sqrt{gy} \]

\[ 1.5 = \sqrt{9.81y} \]

\[ y = 0.23 \text{ m} \]
Design problem:

For this experiment it is necessary to first produce supercritical flow in the flume and then force this flow to become subcritical. The supercritical flow could be produced by means of a sluice gate as shown in Prob. 15.39 and the jump could be forced by means of another sluice gate farther down the flume. Therefore, one needs to include in the design an upstream chamber that will include a sluice gate from which the high velocity flow will be discharged.

The relevant equation for the hydraulic jump is Eq. (15.28). Therefore, to verify this equation \( y_1, y_2 \) and \( V_1 \) will have to be measured or deduced by some other means. A fairly accurate measurement of \( y_2 \) can be made by means of a point gage or piezometer. The depth \( y_1 \) could also be measured in the same way; however, the degree of accuracy of this measurement will be less than for \( y_2 \) because \( y_1 \) is much smaller than \( y_2 \). Perhaps a more accurate measure of \( y_1 \) would be to get an accurate reading of the gate opening of the sluice gate and apply a coefficient of contraction to that reading to get \( y_1 \). The \( C_C \) for a sluice gate could be obtained from the literature.

The velocity, \( V_1 \), which will be needed to compute \( F_r \), can probably be best calculated by Bernoulli’s equation knowing the depth of flow in the chamber upstream of the sluice gate. Therefore, a measurement of that depth must be made.

Note that for use of \( V_1 \) and \( y_1 \) just downstream of the sluice gate, the hydraulic jump will have to start very close to the sluice gate because the depth will increase downstream due to the channel resistance. The jump location may be changed by operation of the downstream sluice gate. Other things that could or should be considered in the design:

A. Choose maximum design discharge. This will be no more than 5 cfs (see Prob. 13.77).
B. Choose reasonable size of chamber upstream of sluice gate. A 10 ft depth would be ample for a good experiment.
C. Choose width, height and length of flume.
D. Work out details of sluice gates and their controls.