Lab #2 Statics: Force on an Inclined Planar Surface
CEE 331 Fall 2004

Laboratory exercise based on an exercise developed by Dr. Monroe Weber-Shirk

Safety

The major safety hazard in this laboratory is a shock hazard. Given that you will be working with water and items running on standard line voltages (the pump and the computer) you should pay attention to the possibility of electric shock. If water spills on the desktop please clean it up if there is no risk of shock. If water gets near a 110 Volt electrical connection DO NOT clean it up. Seek a TA, Professor Brutsaert, or a CEE technician (who have offices across from the lab) for help.

Always work with a minimum of two people.

Objectives

In this laboratory you will measure surface forces acting on an inclined circular plane surface located in a small tank. You will compare measured values of force with theoretical values. You will also investigate the difference between the centroid, the center of pressure, and the line of action of the resultant force.

Theory

Because pressure increases with depth the pressure acting on a submerged surface is a function of depth. The distance from the reference pressure point to the line of action of the resulting force is (make sure you see why!)

\[ y_{RC} = \frac{\gamma \cos \theta}{p_e} \frac{I_{xc}}{A} \]  

1

The second moment of a circle is:

\[ I_{xc} = A \frac{R^2}{4} \]  

2

Substituting equation 1 into equation 2 and solving for the distance along the submerged surface plane between the centroid of the circular surface and the location of the resultant force, we have

\[ y_{RC} = \frac{\gamma \cos \theta}{p_e} \frac{R^2}{4} \]  

3

The distance \( c \) is equal to the distance from the hinge to the centroid of the circular port; the distance \( d \) is equal to the distance from the hinge to the center of pressure, and the distance \( e \) is equal to the distance from the center of pressure to the line of action of the resultant force.

Figure 1. Schematic drawing of the experimental apparatus used to measure surface forces.
to the line of action of the circular port and $y_{RC}$ is the distance from the centroid to the line of action, i.e.,

$$d = c + y_{RC}$$  \hspace{1cm} 4$$

Substituting equation 4 into equation 3 we obtain an equation for the moment arm with respect to the hinge, $d$, based on the geometry and the pressure at the centroid.

$$d = c + \frac{\gamma \cos \theta \cdot R^2}{p_c}$$  \hspace{1cm} 5$$

The force acting on a submerged surface is

$$F_R = p_c A = p_c \pi R^2$$  \hspace{1cm} 6$$

and is the pressure (relative to a reference or datum pressure) acting on the centroid of the surface multiplied by the area of the surface. The moment applied by the water in the enclosed tank on the hinge is

$$F_R d = p_c \pi R^2 d$$  \hspace{1cm} 8$$

The length of the moment arm calculated using equation 5 is based on the moment equations of statics and the pressure exerted by the fluid at the centroid of the circular plane surface. The moment caused by the water pressure in the tank will be transferred by the “y-shaped” aluminum arm to the electronic balance. The electronic balance will give a reading of mass (in grams). A force ($F_w$) can be calculated from the mass by

$$F_w = Mg$$  \hspace{1cm} 9$$

Based on the sum of the moment equation, we know that the moment from the weight must be equal and opposite of the moment from the pressure

$$F_w s = F_R d$$  \hspace{1cm} 10$$

Since the apparatus will be perfectly balanced prior to the experiment, you do not need to worry about the weight of the aluminum arm.

**Experimental Methods**

The apparatus consists of an enclosed tank with a circular port fitted with a circular plug that does not contact the port (i.e., it fits just inside the port) that is covered with a thin membrane (Figure 1). A matching circular port mounted on a hinge supports the membrane. The membrane transmits pressure from the water to the plug. Although the port is hinged its motion is constrained by the load cell and by stops so that it always touches the membrane.

Water can be added or removed from the tank by using the provided pump (which is of centrifugal type). It is important that the centrifugal pump not be operated dry. The pump relies on water for lubrication and the pump will be damaged in about 60 seconds if it is run dry! The pumps’ flow direction is in through the port at the center of the pump head and out through the port located at the circumference of the pump head.
The measured values of the apparatus parameters are given in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_s )</td>
<td>Radius of the plane circular port</td>
<td>3.7 cm</td>
</tr>
<tr>
<td>( c )</td>
<td>Slant distance between the hinge and the center of the port.</td>
<td>16.5 cm</td>
</tr>
<tr>
<td>( b )</td>
<td>Vertical distance between the center of the port and the outside top of the tank</td>
<td>13.82 cm</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Angle between the vertical and the port surface</td>
<td>30 deg</td>
</tr>
<tr>
<td>( s )</td>
<td>Moment arm for balance (distance from hinge to balance)</td>
<td>33 cm</td>
</tr>
<tr>
<td>( e )</td>
<td>Vertical distance between the bench-top and the center of the port</td>
<td>7.78 cm</td>
</tr>
</tbody>
</table>

**General Procedure**

1) Drain the water in the tank using the drain tube that flows directly into the floor drain to below the level of the port by opening the drain valve.

2) Make sure that the arm that rests on the balance is perfectly (as close as you can) vertical. Zero the electronic balance.

3) To fill the tank, first turn the water spigot on to half-way. The spigot is on the green pillar next to the far computer. Next, turn on the valve at the T-junction to half-way. Use the valve next to the tank to fill the tank. Ensure that the “Air Pump Input” tube is *not* connected to the tank, as this will cause the apparatus to explode. At the end of the lab, if the next group is not ready to start, turn off the water supply – it leaks quite a bit.

4) (Trials 2-4 only) For trials requiring pressures other than atmospheric, the peristaltic air pump that was used in Lab 1 will be needed. Attach the air input tube to the port and begin pumping air in or out as needed (be sure the tube “snaps” into place – to remove, press the gray washer towards the tank and pull the tube). Remember, the pump direction can be reversed by pressing the “motor direction” button. If the apparatus leaks air (which it will most likely do), you will need to try to maintain the desired gage pressure by cycling the air pump on/off or running it continuously at a slow flow rate (arrows on the upper left panel of the pump adjust the flow rate up and down). Although you are required to reach “approximate” gage pressures, it is imperative that you record the *actual* gage pressure you reach (i.e. we said approx 5 cm… if you use 6.3 cm, record 6.3 cm).

5) Maintain the water level/air pressure for one minute and record the balance reading.

6) Accurately measure the vertical distance from the centroid of the port to the free surface (i.e., where the water surface is at atmospheric pressure). Note: sometimes the water surface *inside* the tank is *not* at atmospheric pressure!

7) Repeat steps 3 through 7 for trials 1 through 4.
Table 2. Suggested trials.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Water level in tank</th>
<th>Air pressure in tank</th>
<th>$hc$</th>
<th>$d = \frac{r^2 \cos(\theta)}{4h_c} + c$</th>
<th>$d = \frac{F_s s}{\pi h_c \gamma_s^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Top of port</td>
<td>Atmospheric</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Top of port</td>
<td>5 cm of water gage</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5 cm above port</td>
<td>Atmospheric</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5 cm above port</td>
<td>-5 cm of water gage</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Report

1) Calculate the distance between the center of pressure and the centroid for each case based on each method of measuring $d$.
2) Convert your measured water column height $hc$ to the equivalent pressures $pc$.
3) Explain why $d$ changed between the first 2 cases even though the free surface didn’t move.
4) In general as you fill and drain the tank to fill in the table above and cover and uncover the port you will notice the rate at which force changes as measured by the balance relative to the rate water is entering or leaving the tank (when at atmospheric pressure inside). Over the course of your efforts watch this carefully, when is the variation of force linear with depth? When is the variation non-linear with depth? Why! Try filling the tank in 1 cm increments (starting with the water surface just below the port) and recording the force measured. Fill to 5 cm above the top of the port in this manner and then drain the tank in 1 cm increments. Plot your data as an aid to discussing this question. If you see a difference in the force measured on the rising water surface vs. the falling water surface (at the same fill depth) discuss why this might have occurred (this is known as hysteresis).
5) Extra Credit! Develop an equation for calculating $F_s$ as a function of the tank fill level with respect to the bottom elevation of the port (assume the free surface is at atmospheric pressure). Turn in your analytic expression and a plot that covers a depth range of 0 to $4\mathcal{R}$. Does your expression agree with your observations reported in question (4)? (Note you may solve this numerically if you don’t want to play with the tricky calculus!).