Lab #4 Similitude: The Kármán Vortex Street
CEE 331 Fall 2004

Safety
The major safety hazard in this laboratory is a shock hazard. Given that you will be working with water and items running on standard line voltages (the computer) you should pay attention to the possibility of electric shock. The flume has a few very small leaks and there will be a few wet spots under and around the flume. If water gets near a 110-Volt electrical connection DO NOT clean it up. Seek a TA, Professor Brutsaert, or one of the CEE technicians, who have offices across from the lab, for help.

Always work with a minimum of two people.

Objectives
In this laboratory you will investigate the shedding of vortices behind a cylinder where the cylinder axis is perpendicular to the mean flow direction. You will investigate this phenomenon as a function of appropriate dimensionless parameters seeking to experimentally determine the functional form relating the nondimensional shedding frequency with the nondimensional velocity. You will be introduced to a new piece of experimental equipment, the acoustic Doppler velocimeter or ADV. You will also get a chance to explore turbulent flows and get a sense of their unsteadiness.

Theory

The Flow
Perhaps somewhat surprisingly, if you place a cylinder of diameter $D$ in a flow with its axis perpendicular to the mean flow direction an oscillating flow field is established in the wake of the cylinder. This oscillatory wake is known as a Kármán Vortex Street. The only parameters that can be relevant are, $D$, $U$ (the average velocity of the flow across the cylinder), $\rho$, $\mu$, and the shedding frequency $f$. Hence we can write:

$$f = \phi(D, U, \rho, \mu)$$

where $\phi$ is a function of the variables in parenthesis. Now, we have 5 variables and 3 physical dimensions so there must be 2 dimensionless parameters that define the problem. We immediately recognize the Reynolds number as one of them hence all we need is to nondimensionalize the frequency $f$ and we can form a dimensionless relationship. On the back of an envelope we quickly find

$$\Pi_2 = \frac{fD}{U} = \text{Strouhal Number} = St$$

Forming a nondimensional relationship for frequency we have
Aha! All we need to do is conduct experiments at various Re and St, plot the results and we will have found the function $\psi$ relating St to Re.

Now the dimensional analysis we have just conducted assumes an infinitely long cylinder (as the length of the cylinder, $L$, was not included) and a fully submerged cylinder (as $g$ and $\sigma$, gravity and the surface tension, were not included, which are important for free surface flows). Had we included these three additional parameters we would have found three additional $\Pi$’s – the Froude Number, the Weber number, and the aspect ratio $L/D$.

The easiest way to mount cylinders perpendicular to the flow is to mount them vertically and allow them to pierce the free surface. Clearly the depth is not enough to approximate $L/D \sim \infty$. What do we do? Trust the instructor! Equation 4.3 captures the dominant physics even if $L/D \sim 3$ or greater and Fr varies. We will make our measurements in the lower half of the flow depth hence it is reasonable to assume that Weber number will not affect our results. We will take a rough look at these assumptions in our data to verify them.

**Experimental Apparatus**

The experiment will be conducted in the green or yellow flume. The sluice gate has been raised above the anticipated water levels and will not be used for this lab. A taller weir has been fitted to the outlet, which will induce deeper flows (about 30 cm deep depending on flow rate). As in lab #3 the weir will force the outlet flow to be supercritical, which insures that surface wave energy will be forced out of the test section and cannot propagate back into the test section.

The CEE shops have constructed three cylinders with nominal diameters ranging from 2.25 cm to 9 cm. These cylinders mount easily within the flume test section.

The oscillations result in a vortex first being shed from one side of the cylinder and then the reverse side (Figure 1). You can think of this process as the velocity field staying attached to the cylinder longer as it wraps around one side. Hence an excellent indicator of the oscillations will be a strong signal in the $y$ component (across the flume width) of the velocity field, which will oscillate from positive to negative and back to positive through the periodic cycle. Hence the frequency we are interested in is the number of positive OR negative velocity peaks in the $y$ component of velocity in a given time period (NOTE that it is not the number of positive AND negative velocity peaks in the $y$ component of velocity).
We could try to use our Pitot tubes to detect the change in the dynamic velocity head, however, due to the relatively slow response of Pitot tubes, and the relatively small change in dynamic head, it is more reliable to measure the velocity using an acoustic technique.

We will measure the local velocity vector using an Acoustic Doppler Velocimeter (ADV). We will work with an ADV manufactured by Nortek, and known as a Vector or Sontek known simply as a Lab ADV. The details of these ADV can be found at www.nortekusa.com/vec.html and http://www.sontek.com/product/adv/advov.htm, respectively. The Vector ADV is shown in Figure 2. The ADV measures velocity by detecting the Doppler shift in an acoustic pulse transmitted into the water column. The acoustic pulse is transmitted from the center of the metal probe (lower left of Figure 2). The ADV detects the sound scattered by particles and air bubbles in the water column a fixed distance from the probe. It detects the velocity component normal to the three probe tip faces and then through trigonometry the velocity vector in an orthogonal coordinate system can be determined. The three arms are bent at an angle of about 30 degrees to the perpendicular plane of the probe shaft. The result is the 3 acoustic ‘listeners’ are focused to listen at a point about 15 cm below the center of the probe end (where the acoustic ‘speaker’ is located). Hence the ADV measures the velocity at a point 15 cm below the probe end in a cylindrical volume of water that is only about 2.0 cm (10 cm below the Sontek probe) in length and 1.5 cm in diameter. We will use the probe to determine the $x$, $y$ and $z$ components of velocity where we have defined $x$ as positive heading down the flume, $z$ as positive increasing from the flume bottom toward the water surface, and $y$, by the right hand rule is increasing toward the College Town flume wall. More details of the principles of operation for the ADV’s can be found at the above web sites. Note that specific details of working with each ADV will be available in the lab.

**Experimental Methods**

1) Before turning on the flume verify that the water depth is between 17 cm and 24 cm. If you need to add water, use the green hose to fill the open cylindrical storage tank on the left-hand side of the flume. DO NOT OVER FILL THE FLUME (i.e. no more than 24 cm). Should you add too much water there is a drain on the cylindrical storage tank

2) Turn the pump on for your respective flume:

   a. Green Flume: On the backside of the column you encounter when you first enter the lab from the door with the ID card access, you will see a start and stop button in a gray box mounted at head height.
3) Ensure that the sluice gate is above the height of water. DO NOT CHANGE THE HEIGHT OF THE WEIR.

4) The flume will take a few minutes to come to a steady state. You will control the flow rate in the flume in the following manner:

   a. Green Flume: Use the orange valve on the vertical pipe that delivers water into the constant head tank section (it is marked for this lab). DO NOT CLOSE THIS VALVE ALL THE WAY (e.g., do not turn it completely perpendicular to the pipe) as this will cause damage to the pump motor. Leave it wide open to determine $U_{\text{max}}$.

5) Measure the width of the flume.

6) Measure the depth of the water while flowing.

7) You will see that the ADV is set-up in the flume. YOU SHOULD NOT TOUCH THE ADV – you will not need to move it for this lab. You will not be using the usual computers for this lab. Instead, the ADV is directly connected to a computer.

8) READ THE DIRECTIONS POSTED ON THE TABLE ABOUT OPERATING THE ADV SOFTWARE BEFORE GOING BEYOND THIS POINT!!!

9) First, use the ADV to determine the flow velocity without any cylinder in the flume. This will be your Mean Channel Velocity.

![Figure 3. Sample screenshot for $U_{\text{max}}/2$ with large cylinder. You can count about 13 peaks or valleys for Vy (yellow line) in this 30 second time period (note, it has been paused). Therefore, you will need to repeat this “paused” screen at least four times in order to count 50 peaks. Add up the total time to reach 50 peaks when you are calculating your frequency.](image)

10) Insert one of the cylinders into the flow directly in front of the ADV. Your goal is to determine the oscillation frequency, $f$, in the wake of the cylinder. You will do this by counting the number of cross-stream velocity oscillations that occur in a given period of
time (Figure 3). For example, if \( N \) oscillations occur in a time \( T \) then \( f = N/T \). To achieve sufficient accuracy for the experiment you should collect at least 50 oscillations worth of data (e.g., sample for a duration \( T = N/f \)). The following are notes that may help you get good data:

a. You may need to adjust the location of the cylinder (especially for the smallest one) in order to see the oscillations. You’re not measuring the amplitude, just the frequency.

b. Pause the data collection as soon as the time gets to the end of the screen. This will allow you to count all of the peaks over this period. Upon restarting, allow the time to pass through the full range before pausing again to count the peaks. **Only count the peaks OR valleys, not both!!!** You may need to pause several times to reach a total of 50 peaks, especially for the slower oscillations.

c. For very fast frequencies (i.e. smaller diameters and large velocities) you will want to use a smaller time scale so you can clearly see each peak.

d. Make sure you apply the proper velocity scale. For the fast flows, you will probably need a scale of 50 cm/s. For the slower ones, you will need to reduce that to 20 cm/s or lower.

11) Repeat steps 6) thru 10) as necessary for each specified combination of diameter/flow rate listed in the Data Collection/Analysis section.

**Data Collection/Analysis**

Your goal is to determine the function \( \psi \) relating Re to St (equation 4.3). Using the given flow control method for your flume (see Exp. Methods), you should be able to get a maximum flow rate \( (U_{\text{max}}) \) of around 20 – 25 cm/s for the green flume.

You have three cylinders (\( D_1 = 9 \) cm, \( D_2 = 4.5 \) cm \( D_3 = 2.3 \) cm). Varying the speed of flow, make the following measurements:

<table>
<thead>
<tr>
<th>Cylinder Diameter ((D))</th>
<th>Mean Channel Velocity (without the cylinder) ((U))</th>
<th>Depth of Water ((L))</th>
<th>Vortex Shedding Frequency ((f))</th>
<th>( \text{Re}=UD/\nu )</th>
<th>( \psi=fD/U )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_1 = )</td>
<td>( U_{\text{max}} = )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D_2 = )</td>
<td>( U_{\text{max}} = )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D_3 = )</td>
<td>( U_{\text{max}} = )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D_1 = )</td>
<td>( U_{\text{max}}/2 = )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D_2 = )</td>
<td>( U_{\text{max}}/2 = )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D_3 = )</td>
<td>( U_{\text{max}}/2 = )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D_1 = )</td>
<td>( U_{\text{max}}/4 = )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D_2 = )</td>
<td>( U_{\text{max}}/4 = )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D_3 = )</td>
<td>( U_{\text{max}}/4 = )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D_1 = )</td>
<td>( U_{\text{slow}} = )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D_2 = )</td>
<td>( U_{\text{slow}} = )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D_3 = )</td>
<td>( U_{\text{slow}} = )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
$U_{\text{max}}$ is the maximum velocity you can achieve in your flume. Once you determine that, make your measurements for each of the three cylinders. Next, reduce the Mean Channel Velocity to roughly $U_{\text{max}}/2$. You will then repeat the measurements for each cylinder. Reduce the velocity once more to roughly $U_{\text{max}}/4$ and repeat once again. For the $U_{\text{slow}}$ measurements, slow the velocity down to < 5 cm/s.

**Laboratory Report**

Submit a report that includes the following:

1. A plot that shows your measured St as a function of Re (hence a plot of $\psi$).
2. You should notice that three of your Re values are very similar. My guess is you will find that your three measurements at constant Re do not collapse too well (i.e., they do not have very similar St numbers). In fact I bet a trend exists that shows larger cylinders have larger St. What happened? Did dimensional analysis fail? Is my suggestion that Fr, We, and $L/D$ effects are minor wrong? Recall that we said nothing about walls or the width of the channel in which we mounted the cylinder – hence we implicitly assumed an unbounded domain. In reality conservation of mass requires that as the flow is blocked by the cylinder the mean velocity of the flow going around the cylinder must be higher than the measured $U$. Aha, we see that $St \propto U^{-1}$. It looks like if we recalculate the St number (and Re) based on $U_L$, the local mean velocity around the cylinder, we will get a better collapse. Using conservation of mass determine $U_L$ (the flume is too narrow to get the ADV into this region) and re-plot your data. Does it look better?
3. Calculate the $L/D$ ratio and the Froude Number for each measurement. For your experiments at constant Re is there a significant variation with $L/D$ ratio or Fr?